– Dimensions and Units :-

	Dimension	Unit
1.	Length	meter (m)
2.	mass	kilogram (kg)
3.	Time	second (s)
4.	Temperature	kelvin (K)
5.	Electric current	Ampere (A)
6.	Amount of Light	Candela (c)
7.	Amount of matter	mole (mol)

Quantity	SI Units	From English to SI units
Area	m^2	1 ft = 0.3048 m
Density	kg/m^3	1 in = 0.0254 m
Energy	J	1 lbm = 0.4536 kg
		1 Slug = 14.594 kg
Force	N	1 lb_f = 4.448 N
Heat flux	W/m^2	
Heat transfer rate	W	1 Btu = 1055.06 J
		1 Cal = 4.1868 J
		1 Btu/lbm = 2326 J/kg

Quantity	SI Units	from English to SI Unit
mass flow rate	kg/s	
Power	W (J/s)	1 hp = 745.7 W
Pressure & Stress	N/m^2	$1\ lb_f/ft^2 = 47.88\ N/m^2$ $1\ psi = 1\ lb_f/in^2 = 6894.8\ \frac{N}{m^2}$
Specific heat	J/kg.k	
Surface tension	N/m	
Velocity	m/s	
Viscosity	m^2/s	
Volume	m^3	
Volume flow rate	m^3/s	
Specific volume	m^3/kg	

1 Standard atmospheric pressure $= 1.01325 \times 10^5\ \frac{N}{m^2}$

$1\ N/m^2 = 1$ Paskal $= 1$ Pa

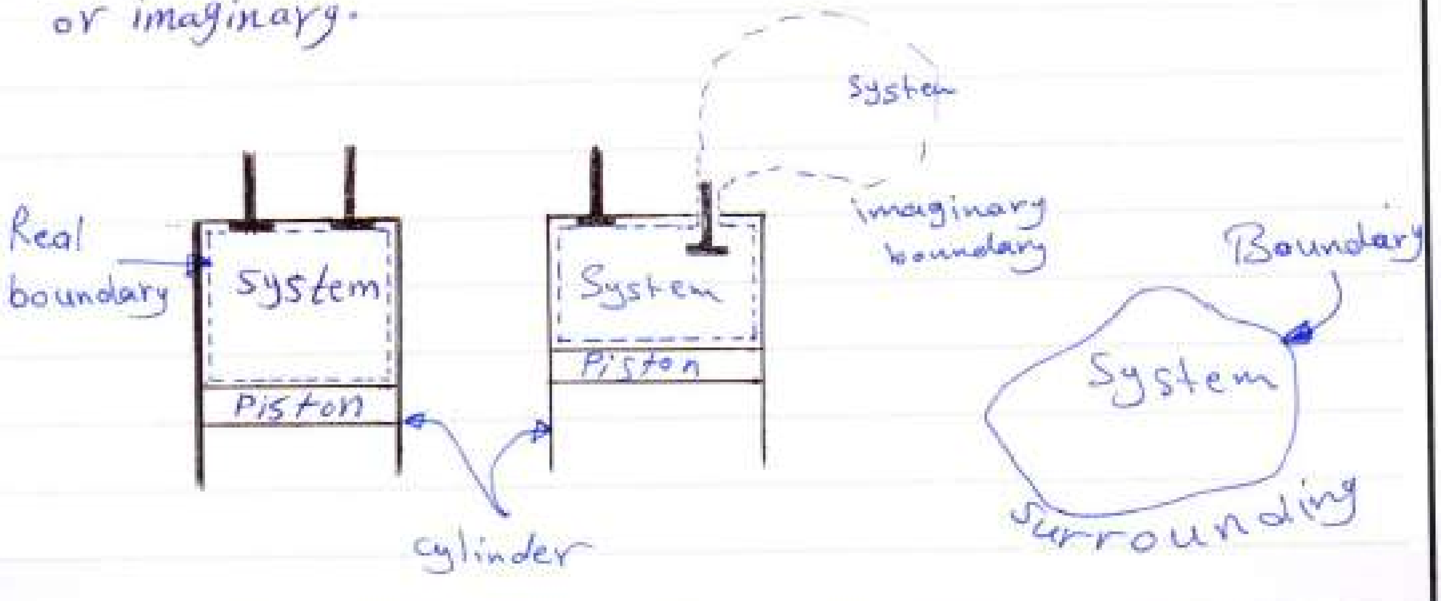
$1\ gal = 0.003785\ m^3$

- **Thermodynamics :-**

Thermodynamic is an axiomatic science which deals with the relations among heat & Work and Properties of system which are in equilibrium. It described state and changes in state of Physical system.

— **System :-** A system is a finite quantity of matter.

— **Boundary :-** The actual envelope enclosing the system is the boundary of the system. The boundary may be fixed or it may move. The boundary may be real or imaginary.



- Working Substance :-

The working substances are in general fluids which are capable of deformation in that they can readily be expanded and compressed. Common examples of working substances used in thermodynamic system air and steam.

- Pure Substance :-

Is a single substance or mixture of substance which has the same consistent composition throughout such as water, Nitrogen, Heli-

- Heat , Work :-

Heat is a form of energy which is transferred from one body to another body at lower temperature by virtue of the temperature difference between the bodies. The symbol of heat is "Q" .

Work is defined as the product of the force (F) and the distance moved (x) in the direction of the force.

$$\text{Work} = \text{Force} \times \text{distance}$$

Note: The heat and work are ~~a~~ transient quantites not properties.

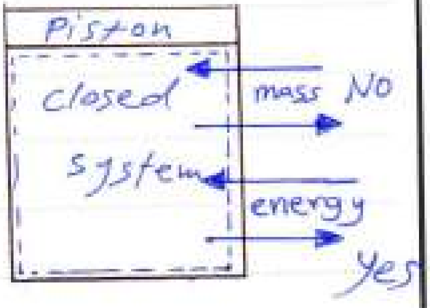
– Closed and Open system:-

A system may be considered to be closed or open depending on whether a fixed mass or a fixed Volume in space is chosen for study.

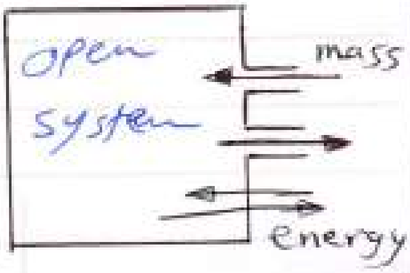
A closed system Consist of a fixed amount of mass and no mass Can across its boundary, that is no mass can enter or leave a closed system.

But energy in the form of heat or work can cross the boundary. and the Volume of a closed system does not have to be fixed.

If as a special Case, even energy is not allowed to cross boundary, that system is Called an Isolated system.



An Open system, It involves mass flow, such as a compressor, turbine or nozzle. Both mass and energy Can cross the boundary.



Adiabatic System :- ($Q=0$)

Is one which is thermally insulated from its surrounding, it can exchange work with its surrounding.

- Properties of a System :-

Any characteristic of a system is called a property. Some familiar examples are Pressure (P), temperature (T), Volume (V) and mass (m).

Note all properties are independent, some are defined in terms of other ones. such as density ρ, specific volume v, specific gravity ρ_s.

Properties are considered to be either intensive or extensive.

Intensive properties are those that are independent of size of a system. Such as temperature, Pressure and density.

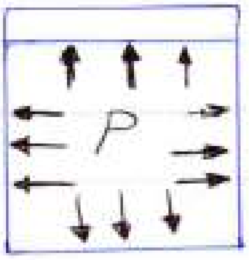
Extensive Properties are those whose values depend on the size or extent of the system. Such as mass, Volume, and total energy.

- Pressure :-

Pressure is defined as a force per unit area.

has the unit of Newtons per square meter (N/m²) which called a pascal (Pa).

$$\boxed{P = F/A}$$



$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$

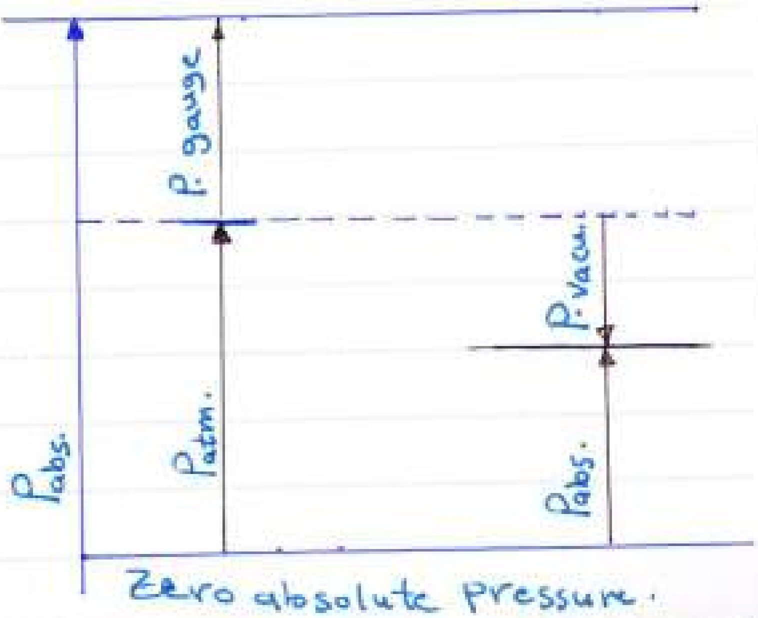
$1 \text{ Patm.} = 1.01325 \times 10^2 \text{ kPa} = 1.01325 \text{ bars}.$

$1 \text{ atm} = 14.696 \text{ psi} = 760 \text{ mm Hg}.$

The difference between the absolute pressure and the local atmospheric pressure is called gauge pressure. Pressure below atmospheric pressure are called vacuum pressure.

$P_{gauge} = P_{abs.} - P_{atm.}$

$P_{vacu.} = P_{atm.} - P_{abs.}$



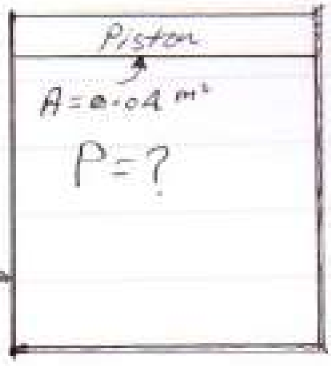
Example :- A vacuum gage connected to a chamber reads 5.8 psi at location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

Sol.

$$P_{abs.} = P_{atm.} - P_{vac.} = 14.5 - 5.8 = 8.7 \text{ Psi}.$$

Example :- The piston of a piston-cylinder device containing a gas has mass of 60 kg and a cross-section area of 0.04 m² as shown below. The local atmospheric pressure is 0.97 bar and the gravitational acceleration is 9.81 m/s². a) Determine the pressure inside the cylinder.

Sol.



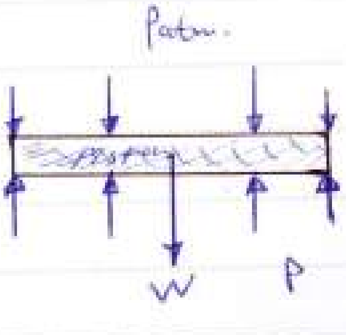
Equilibrium $\Rightarrow \Sigma F = 0$

$$P * A = P_{atm.} * A + W$$

$$P = P_{atm} + \frac{W}{A} = P_{atm} + \frac{m \cdot g}{A}$$

$$= 0.97 + \frac{60 * 9.81}{0.04} = 1.117 \text{ bars}.$$

b) If some heat is transferred to the gas and its volume doubles, do you expect the pressure inside the cylinder to change



– Temperature :-

The temperature is define as measure of hotness.

– Temperature Scales :-

The temperatere scales used in the SI Units and in English units system are the celsius scale « centigrade scale » and the «Fahernheit scale».

The temperature scales were based on two fixed Points

i – The Steam Point « Boiling Point » of the water at standard atmospheric Pressure.

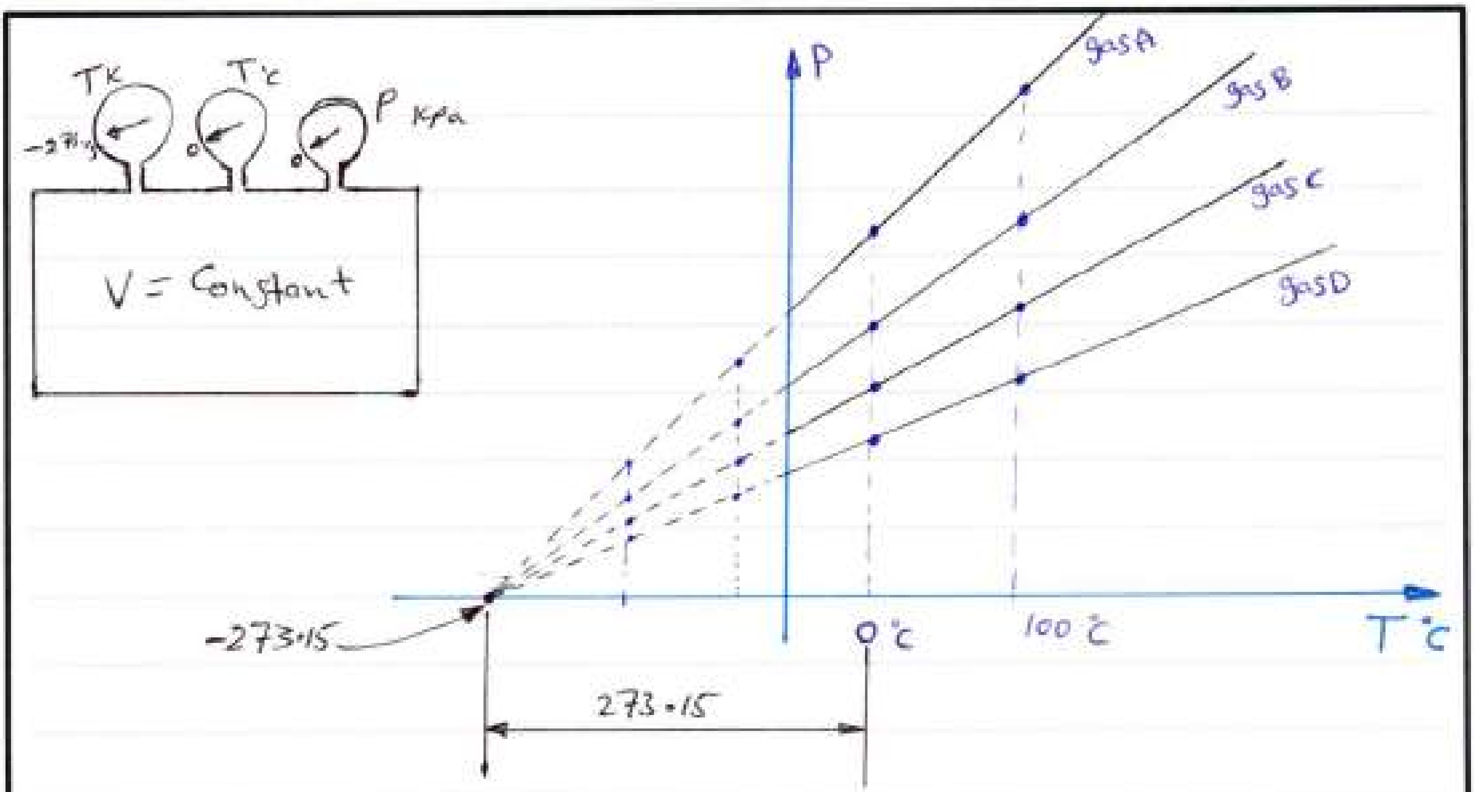
ii – The ice Point « freezing point » of the water.

The fixed Points for these temperature scales are

Temperature	Celsius Scale	Fahernhiet scale
Steam Point	100	212
Ice Point	0	32
Interval	100	180

$$^\circ C = \frac{5}{9}(F-32)$$

$$K = {}^\circ C + 273 \quad ; \quad R = 1.8 * K = F + 460$$



The temperature scale in thermodynamic, In SI Unity system Called Kelvin Scale (K) and in English System is the Rankine (R).

[Pressure Versus Temperature at experimental data obtained from a Constant Volume gas thermometer using four different gases at different Pressure].

$$T(K) = T(°C) + 273.15$$

- State and Equilibrium :-

Consider a system that is not undergoing change. At this point all Properties can be measured or Calculated throughout the entire system, which gives as a set of properties that completely describe the Condition, or the state of the system.

A system at two different states

a. State 1 b. State 2

thermodynamics deals with equilibrium states - The word Equilibrium implies a state of balance.

- Types of Equilibrium.

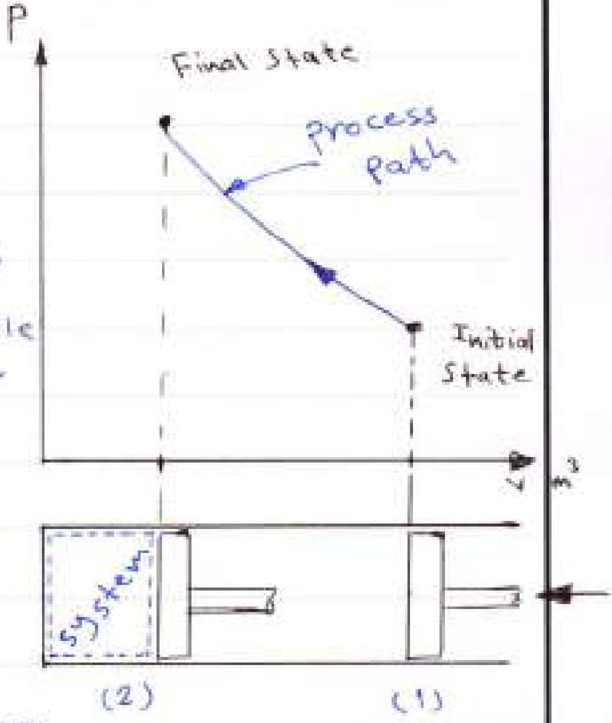
1. Thermal equilibrium:- The temperature of the system does not change with time and has same value at all points of the system.
2. Chemical equilibrium:- No chemical reaction take place in the system and the chemical composition which is same throughout the system does not vary with time.

- Processes and cycles :-

* State :- It's the condition of the system at an instant of time described by it's properties.

Any change that a system undergoes from one equilibrium state to another is called a Process, and the series of states through which a system passes during a process is called the path of the process.

Some common properties that are used as coordinates are temperature T, Pressure P, and Volume V, or specific volume v.



The prefix iso- is often used to designate a process for which a particular property remains Constant.

An isothermal process, for example is a process during which the temp. T remains Constant.

An isobaric process is a Process during which the Pressure P remains Constant.

An isochoric (or isometric) Process is a Process during which the specific Volume v remains Constant.

- Types of thermal Cycle :-

1- Thermodynamic cycles:

In this cycle no change in the chemical structure and the properties of fluid for example the Steam in power plant.

2- Mechanical Cycles:

In this cycle the properties of fluid or Substance will be change for example the Internal-Combustion Engine

- Energy :-

Energy is defined as that capacity a body or Substance possesses which can result in the Performance of Work. From the Law of Conservation of energy the energy cannot be relativelly created or destroyed.

- Forms of Energy :-

Energy can exist in numerous forms such as thermal, mechanical, kinetic, Potential, electric, chemical, and their Sum Constitutes the total energy E of a system.

1. Kinetic energy:- The energy that a system possesses as a result of its motion relative to reference is called Kinetic Energy ((K.E.))

$dE = F \cdot ds$

$F = m \cdot a = m \cdot \frac{dV}{dt}$

V: velocity , t: time

$\therefore dE = m \frac{dV}{dt} \cdot ds = m \frac{ds}{dt} \cdot dV$, $\frac{ds}{dt} = V$

$\therefore \int dE = \int m \cdot V \, dV \Rightarrow \int_0^E dE = m \int_{V_1}^{V_2} V \, dV$

$\therefore E = m \frac{V^2}{2} \Big|_{V_1}^{V_2} = \frac{1}{2} m (V_2^2 - V_1^2)$. if $V_1 = 0$

$\therefore \boxed{K.E = \frac{1}{2} m V_2^2}$ kJ

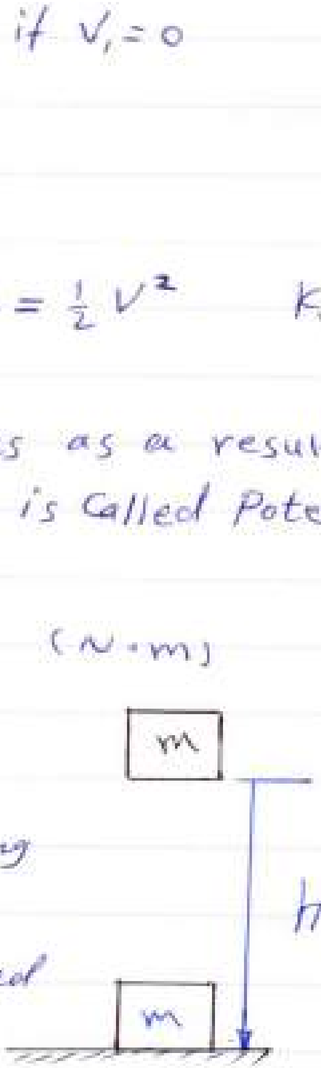
or for unit mass « specific k.E » $k.e = \frac{1}{2} V^2$ kJ/kg

2- The energy that a system possesses as a result of its elevation in a gravitational field is called Potential Energy P.E and is expressed as:

$P.E = W \cdot h = m \cdot g \cdot h$ «J» (N.m)

g: gravitational acceleration

h: elevation



Any volume of fluid entering or leaving a system displace an equal volume, the energy produced due to this flow is called Flow energy $FE = P \cdot V$

3- Heat Energy :-

It is one form of energy produced when Temperature difference between the system and Surrounding. The symbol is "Q".

4- Work Energy :-

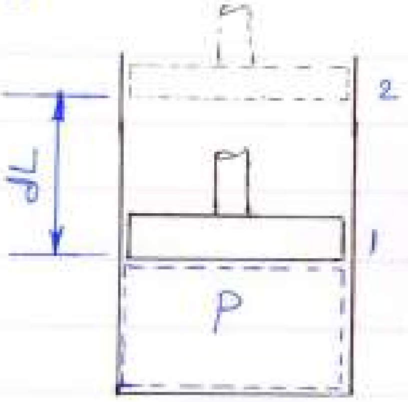
It is a form of energy defined as the multiplication of the force effected on the mass by the distance that the mass will be moved due to this effect.

$$dW = F \cdot dL = P \cdot A\,dL \quad , \quad dV = A\,dL$$

$$dW = P \cdot dV$$

$$\therefore \int_1^2 dW = \int_{V_1}^{V_2} P\,dV \Rightarrow$$

$$\boxed{W_{1-2} = \int_{V_1}^{V_2} P \cdot dV} \quad kJ$$



Power : Is the rate of doing work

$$P = \frac{W}{t} \quad J/s \text{ "Watt"}$$

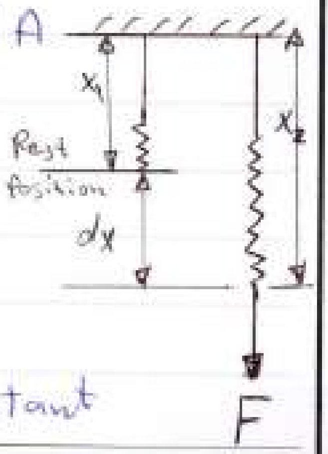
$$P = W \cdot \dot{m} \quad , \quad \dot{m} = \text{flow rate} = \rho V A$$

- Spring Work:

$$dW = F \cdot dx \quad , \quad F = k \cdot X$$

$$\int dW = \int_{x_1}^{x_2} k\,x\,dx \Rightarrow W = k\int_{x_1}^{x_2} x\,dx = \frac{k}{2}(x_2^2 - x_1^2)$$

$$W_{spring} = \frac{1}{2}k(X_2^2 - X_1^2) \; ; \; k: \text{Spring Constant}$$



The total energy of a System in two groups :-
1. Microscopic 2. Macroscopéc

5- The microscopic forms of energy are those related to the molecular structure of a system. The Sum of the microscopic forms of energy is called the (Internal Energy) of a system and is denoted by (U).

Ex. : An insulated 2 kg box falls from a ballon 3.5 km above the earth. what's the change in Potential energy of the box after it has hit the earth Surface.

Sol. : $P.E = m.g\,h = 2 * 9.81 * 3500 = 6867$ J

Ex. : A force (F) is proportional to (x^2) and has the value of 133 N when $x = 2$, Determine the work done by it moves in object from $x = 1$ to $x = 4$ where (x) in meter.

Sol. : $w = \int_{x_1}^{x_2} F \cdot dx = \int_1^4 k x^2 dx = k, \int_1^4 x^2 dx$

$F \propto x^2$
$F = k\,x^2$

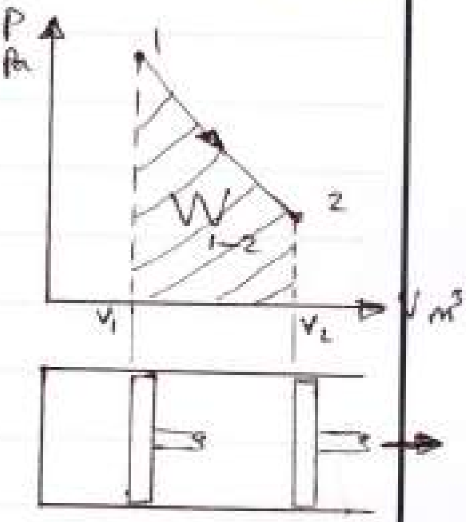
$$w = k\left[\frac{x^3}{3}\right]_1^4 = \frac{133}{4*3}\left[4^3 - 1^3\right] = 69825 \text{ N.m}$$

$F = k\,x^2$
$133 = k * (2)^2$
$\boxed{k = \frac{133}{4}}$

Ex.: Let the Pressure in the cylinder in the fig. given by the equation $P = C/V$ as a function of Volume.

a) find the work done, if the initial pressure is 400 kPa the initial volume $0.02\ m^3$ and the final Volume is $0.08\ m^3$

b) Is the sign correct?



Sol.:-

a) $P = \frac{C}{V} \Rightarrow PV = C$

$\therefore P_1V_1 = P_2V_2 = \text{Constant}$

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{C}{V}\,dV$$

$$= C\int_{V_1}^{V_2} \frac{dV}{V} = P_1V_1\int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = P_1V_1 \ln\frac{V_2}{V_1} = 400 \times 0.002 \times \ln\left(\frac{0.08}{0.02}\right) = 11.09\ kJ$$

b) the Sign Correct Since it is Positive and the work is done by the system.

Ex.: The pressure in the cylinder "as Shown" Varies the following manner with Volume $P = C/V^2$, if the initial Volume is $0.05\ m^3$ and the final Pressure 200 kPa find the work done by the system. the initial Pressure is 500 kPa.

- Reversibility:

A more rigorous definition of reversibility is follows:

When a fluid undergoes a reversible process, both fluid and its surrounding can be restored to their original state.

The criteria of reversibility are as follows:-

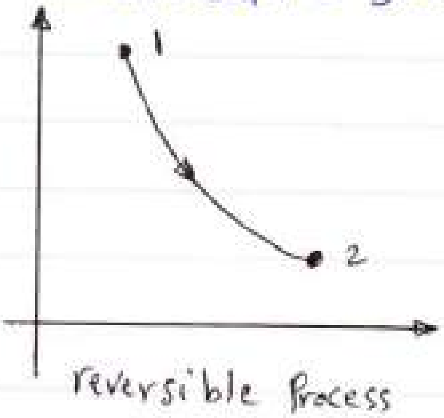
1- The Process must be frictionless.

2- The difference in Pressure between the fluid and its surrounding during the Process must be infinitely small.

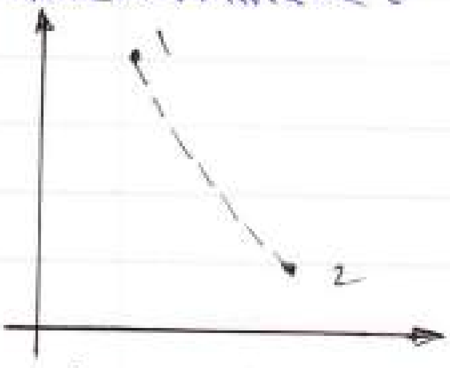
3- The difference in temperature between the fluid and its surrounding during the process must be infinitely small.

A reversible process between two states drawn a line on diagram of properties.

An irreversible process is represented by a dotted line joining the end states to indicate the intermediate states are indeterminate.



reversible Process



irreversible Process

- Internal reversibility :- It is may be obtained, because no process in practice is truly reversible, but with conditions below :-

1. The surrounding can never be restored to their original state. and the path of the process
2. The fluid itself is at all times in an equilibrium state and the path of the process can be exactly retraced to the initial state.

Reversibility or Reversible Process

هي العملية التي يعود فيها النظام إلى حالته الاصلية. فإذا اجري اجراء ما على النظام، ثم اوقف هذا الاجراء وعكس اتجاهه بحيث عادت كميات الشغل والحرارة إلى النظام والمحيط كل إلى مصدره الاصلي، فان هذا الاجراء انعكاسي.

فمثلا نتصور تمدد المائع ادياباتيا في توربين، ثم يضغط في ضاغط، فإذا تم ذلك بحيث يعود المائع إلى حالته الميكانيكية والثرموديناميكية الاصلية، وبحيث يكون الشغل الناتج عن التوربين كافيا لتشغيل الضاغط، عندئذ يكون التمدد في التوربين محققا لفكرة الاجراء الانعكاسي. وبنفس الطريقة من التمدد نظريا أن يعاد المائع المار في منفث إلى حالته الابتدائية اذا حررناه في ناشر من دون تأثير في المحيط.

ان معظم العمليات الحقيقية تتم داخل نظام يتألف من مكبس واسطوانة، كمكائن الاحتراق الداخلي الترددية، الضواغط الغازية ... الخ. ولغرض التبسيط نفترض ان العمليات فيها انعكاسية. التي يمكن

الحصول عليها بدفع المكبس ببطء جداً لتجنب الاضطراب وكي يكون هناك وقت كافٍ ليصبح الضغط ودرجة الحرارة متساوياً في جميع النقاط.

العملية الانعكاسية مثالية افتراضية، أي لا توجد في العمليات الطبيعية أو الدورات الحقيقية. ولكن يمكن التقرب من هذه العمليات عند توفر الشروط الافتراضية التالية:

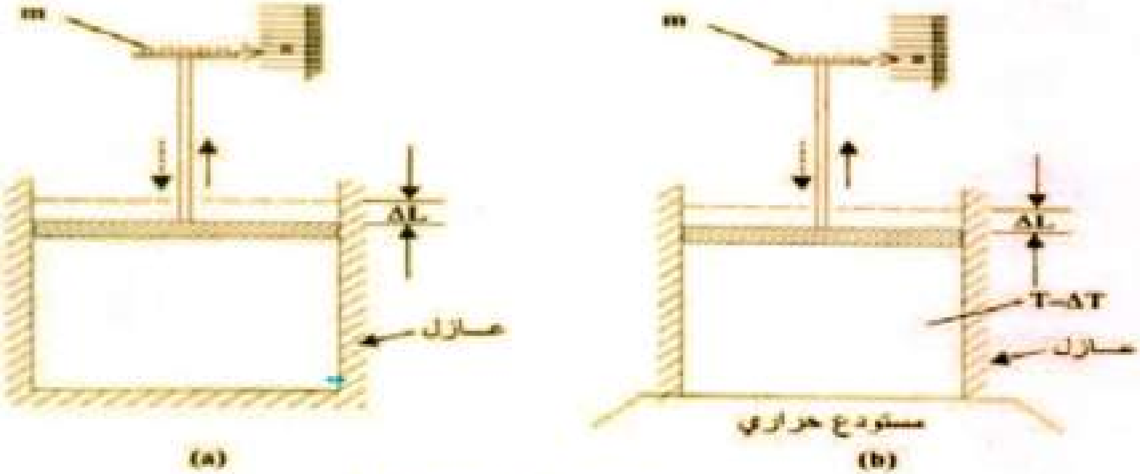
1. **أن تكون العملية بطيئة. وهذا يؤدي إلى:**
   أ. انعدام الاحتكاك الميكانيكي والمائعي.
   ب. تساوي القوى المؤثرة على المكبس.
   جـ .انعدام الفقدان الحراري.
2. **ان تكون العملية متزنة حراريا. وهذا يؤدي إلى:**
   أ. تساوي ضغط ودرجة حرار النظام والمحيط.
   ب. انتقال الشغل والحرارة من وإلى النظام متساوي في الاتجاهين.

فمثلا في عملية تمدد الغاز ضمن اسطوانة مجهزة بمكبس، يوجد مقدار قليل من الاحتكاك وفرق قليل في ضغط ودرجة حرارة النظام والمحيط بحيث يمكن اهمالها. أي أن المكبس في هذه الحالة يكون عديم الوزن والاحتكاك والتسرب. وبالتالي يمكن حساب الشغل على اساس أن التمدد انعكاسي.

الاجراء اللاانعكاسي Irreversible Process

هي العملية التي تحدث بإتجاه واحد و لا يمكن عكسها. أي لا يمكن أن يعود النظام إلى حالته الاصلية تلقائيا أو دون تأثير خارجي، بسبب وجود الاحتكاك وانعدام الاتزان الحراري.

ولمناقشة هذه العملية نفترض وجود اسطوانة معزولة تحتوي على غاز مغلق بمكبس معزول فوقه اثقال. ونفترض ايضا أن المكبس يلتصق كثيراً بجدران الاسطوانة.



الانعكاسية واللاانعكاسية

عند إزالة ثقل من هذه الاثقال فيتحرك المكبس مسافة (ΔL) حاملا معه بقية الاثقال. أي أن النظام ينجز شغلا على الخليط. وأثناء هذه العملية يتسارع المكبس في البداية ثم يتباطأ تدريجيا حتى يصل إلى الحالة النهائية. وقد يتذبذب المكبس بين الحالتين ولكن هذه الحركة التذبذبية تخمد بالتدريج نتيجة للاحتكاك الميكانيكي والمائعي.

هذه التأثيرات تمنع وصول المكبس إلى الارتفاع الذي يمكن الوصول إليه في غيابها، لذلك يقل الشغل المنجز. هذه العملية لا انعكاسية لأنها مصحوبة بمفقودات حرارية جراء الاحتكاك. وتبقى العملية لا انعكاسية حتى في حالة انعدام الاحتكاك الميكانيكي وبقاء الاحتكاك المائعي. وبما أن هذه الظواهر موجودة دائما لذا فأن الاجراءات الحقيقية لا يمكن أن تكون انعكاسية.

- Reversible work :-

Consider an ideal frictionless fluid contained in a cylinder behind a piston, with the following assumptions:-

1. the pressure and temperature of the fluid are uniform.
2. no friction between the piston and the cylinder wall.

work done by fluid = $F * dL$

$F = P * A \Rightarrow W = P * A * dL = Pdv$

per unit mass

$W = P\,dv$

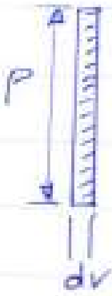
where (v) is the specific volume

$dw = F * dL$

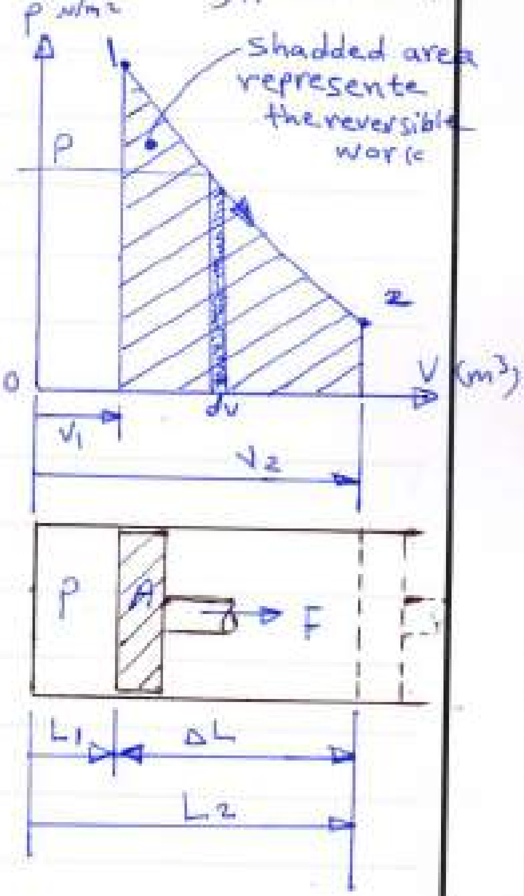
$F = P * A$

P: Pressure in the cylinder

A: Cross-section area of the piston.



The reversible work = the area under the curve on (P-V) diagram.


Shadded area represente the reversible work

$$dw = P * A * dL \quad ; \quad A * dL = dV$$

$$\therefore dw = P * dV \Longrightarrow W = \int_{V_1}^{V_2} P * dV = P(V_2 - V_1)$$

Note: Irreversible work $\neq \int_{V_1}^{V_2} P\,dV$.

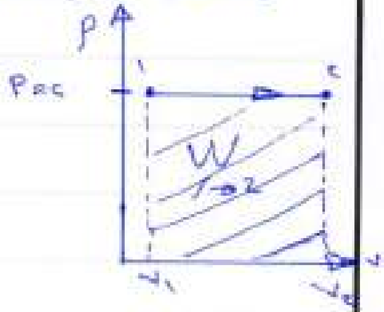
Ex. A certain fluid at 10 bars is contained in a cylinder behind a piston, the initial volume being 0.05 m^3. Calculate the work done by the fluid when it expands reversibly.

a- at constant pressure to final volume of 0.2 m^3.

Sol.

$$\text{Reversible work} = \int_{v_1}^{v_2} P * dv$$

$$\therefore W = \int_{0.05}^{0.2} 10\times10^2\, dv = 10^3 * (0.2 - 0.05) = 150\ kJ$$



(a)

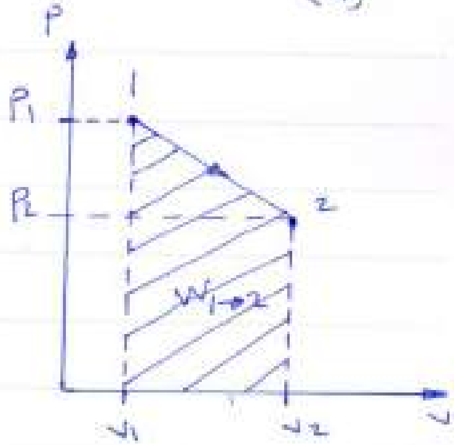
b- According to a linear law to a final volume of 0.2 m^3 and final pressure of 2 bars.

$$* \text{ linear law} = \frac{P - P_1}{V - V_1} = \frac{P_2 - P_1}{V_2 - V_1}$$

$$\frac{P - 10\times10^2}{V - 0.05} = \frac{2\times10^2 - 10\times10^2}{0.2 - 0.05}$$

$$0.15P + 800V - 190 = 0$$

$$\therefore P = \frac{190 - 800V}{0.15}$$

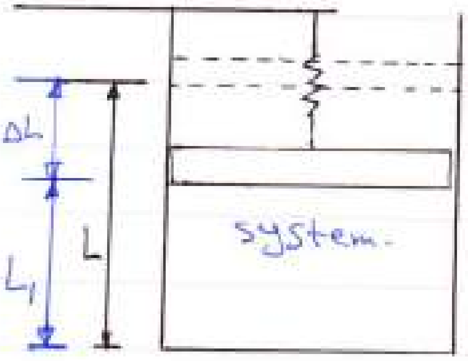


$$W = \int_{v_1}^{v_2} P * dv = \int_{0.05}^{0.2} \left(\frac{190 - 800V}{0.15}\right) dv = \frac{1}{0.15}\left[190V - 400V^2\right]_{0.05}^{0.2}$$

$$\therefore W = 90\ kJ$$

Ex.: Consider the system shown in fig., the initial volume inside the cylinder is $0.1\ m^3$. At this state the pressure inside is 100 kpa. which just balance the atmosphere pressure plus the Piston weight, the spring is touching but exerts no force on the Piston at this state. The gas now heated until the volume is doubled. The final Pressure of the gas is 300 kpa and during the Process the spring force is proportional to the displacement of the Piston from the initial position. Calculate the work done by the system.

SoL.:-



$P_1 = 100\ kpa$, $V_1 = 0.1\ m^3$

$P_2 = 300\ kpa$, $V_2 = 0.2\ m^3$

$F \propto \Delta L$, $\Delta L = L - L_1$

$F_s \propto (L - L_1)$

$(F_s/A) \propto (L - L_1)/A$

$$P_s \propto \frac{(L-L_1)A}{A^2} \Rightarrow P_s \propto \frac{V-V_1}{A^2}$$

$$P_s = \frac{C}{A^2}(V - V_1) = a(V - V_1)$$

$$P_t = P_0 + P_s = 10^2 + a(V_2 - V_1) \Rightarrow 300 = 10^2 + a(0.2 - 0.1) \Rightarrow a = 2000$$

$$P_t = 10^2 + 2000(V - 0.1) = 2000V - 100 \Rightarrow W = \int_1^2 P\,dV = \int (2000V - 10^2)\,dV$$

$$W_{12} = \left[\frac{2000V^2}{2} - 10^2 V\right]_{0.1}^{0.2} = 20\ kJ.$$

If there is no spring the Piston will rise at constant pressure.

$$W = \int_1^2 P\,dV = P\int_1^2 dV = P(V_2 - V_1) = 10^2(0.2 - 0.1) = 10\ kJ$$

c - According to Law $PV = c$ to final volume of $0.1\ m^3$.

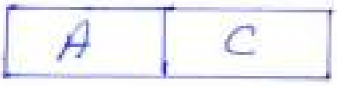
d - According to Law $PV^3 = c$ to a final Volume of $0.06\ m^3$

e - According to Law $P = (a/V^2) - (b/V)$ to a final volume of $0.1\ m^3$ and final Pressure of 1 bar. (a & b) are constant

- The Zeroth Law :-

This Law is Concerned with thermal equilibrium. It states that if two bodies are separately in thermal equilibrium with a third body then they must be in thermal equilibrium with each other.

thermal equili.



thermal equili.

thermal equili.

If $T_A = T_B$, $T_A = T_C \Rightarrow \therefore T_B = T_C$ (thermal equilibrium).

- The first Law of thermodynamics :-

° When a system undergoes a thermodynamic cycle then the net heat supplied to the system from its surrounding is equal to the net work done by the system on its surrounding.

$$\oint dQ = \oint dW$$

where $\oint$ represents the Sum for a complete cycle.

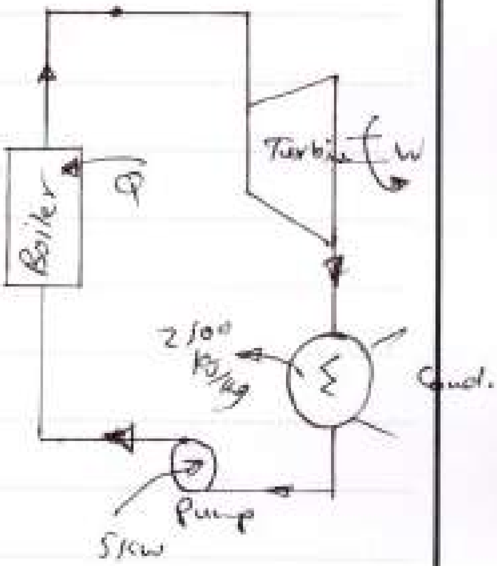
Ex.: 1.5 $\frac{kg}{s}$ Steam flow rate in a steam power Plant where the turbine develops 10^3 $\frac{kJ}{s}$, the heat rejected by the system to cooling water in the Condenser is 2100 kJ/kg and the feed pump work required to pump the Condensate back into the boiler is 5 kw. Calculate the heat supplied to the Steam in the boiler.

Sol.

$\Sigma dQ = \Sigma dw$

$\Sigma dw = 10^3 - 5 = 995$ kw

$Q = \dot{m} * q$



from first law of thermo.

$\Sigma Q = \Sigma W$

$\Sigma \dot{m} q = \Sigma W$

$1.5 * q = 995$

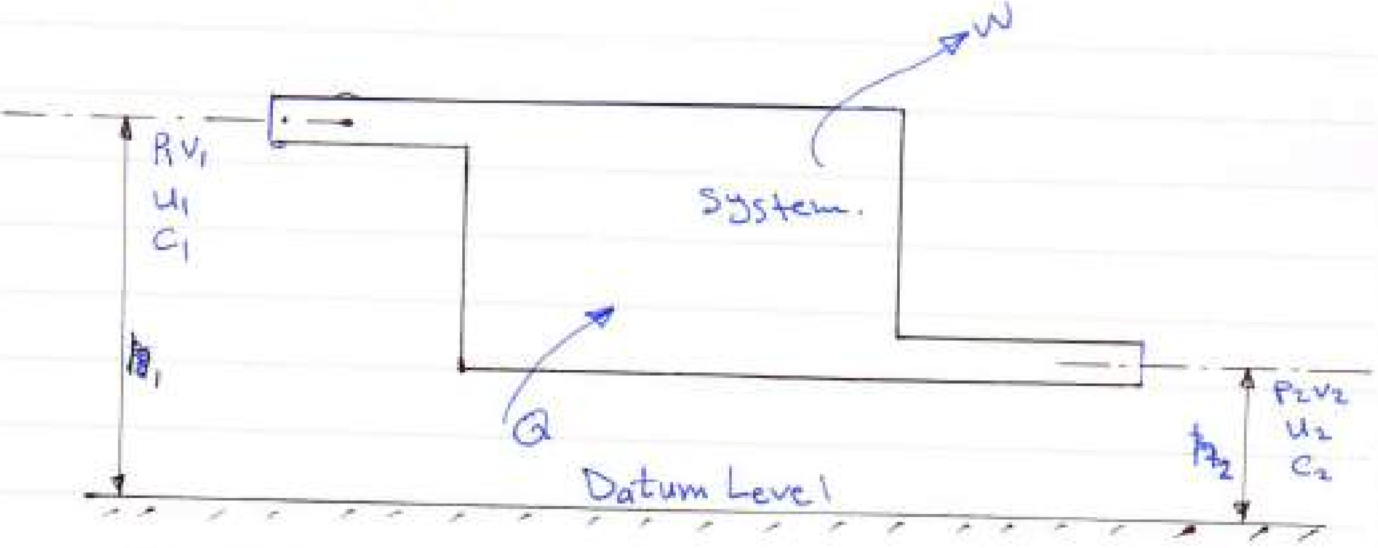
$\therefore q =$

- The Steady flow energy equation:- (SFEE).

This equation is a mathematical statement of Principle of Conservation of energy as applied to the flow of a fluid through a thermodynamic system.

The Various forms of energy which the fluid can have are as follows:-

1. Potential energy : $P.E = mgZ$ kJ
2. kinetic energy : $K.E = \frac{1}{2} m C^2$ kJ
3. Internal energy : $U = m.u$ kJ
4. Flow or displacement energy : $F.E = P.V$ kJ
5. Heat received or rejected : Q
6. External work done : W



Applying the principle of conservation of energy to the system, then:

Total energy entering the system = Total energy leaving the system.

$$E_{input} = E_{output}$$

$$P.E_1 + I.E_1 + F.E_1 + K.E_1 + Q = P.E_2 + I.E_2 + F.E_2 + K.E_2 + W$$

for 1 kg of fluid:

$$gz_1 + u_1 + P_1v_1 + \frac{1}{2}C_1^2 + q = gz_2 + u_2 + P_2v_2 + \frac{1}{2}C_2^2 + w$$

$$gz_1 + (u_1 + P_1v_1) + \frac{1}{2}C_1^2 + q = gz_2 + (u_2 + P_2v_2) + \frac{1}{2}C_2^2 + w.$$

$u_1 + P_1v_1 = h_1$ The enthalpy ; $u_2 + P_2v_2 = h_2$

$$\boxed{gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w}$$

Steady - Flow Energy Equation (SFEE)

where q & w per unit mass

Mass flow rate: $\dot{m} = \rho C A = \frac{CA}{v}$.

- The non-flow energy equation: (NFEE)

In the case of a closed system, however in which the fluid mass remains constant, no substance passing through the system boundary, the flow terms in study flow E.E. will not apply. Thus the terms PV and $C^2/2$ are neglected. The system is then said to be non-flow.

from the SFEE

$$\cancel{gz_1} + u_1 + \cancel{P_1V_1} + \cancel{\frac{1}{2}c_1^2} + q = \cancel{gz_2} + u_2 + \cancel{P_2V_2} + \cancel{\frac{1}{2}c_2^2} + w$$

$$u_1 + q = u_2 + w \Rightarrow q - w = u_2 - u_1$$

when $\Delta z \simeq 0$, $\Delta c^2 \simeq 0$

∴ $\boxed{q - w = u_2 - u_1}$ NFEE

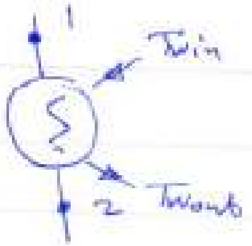
$\boxed{q - w = h_2 - h_1}$ SFEE

- Some Applications of the SFEE :-

1. Steam Boiler : from SFEE ,
   in Boiler , $w = 0$, $c_1 \simeq c_2$, $\Delta z \simeq 0$
   ∴ $q = h_2 - h_1$ kJ/kg

2. Turbine ; from SFEE.
   in turbine $Q = 0$, $(c_2^2 - c_1^2/2) \simeq 0$, $\Delta z \simeq 0$
   ∴ $w = h_1 - h_2$ kJ/kg

3. Heat Exchanger: from SFEE.



in h.e

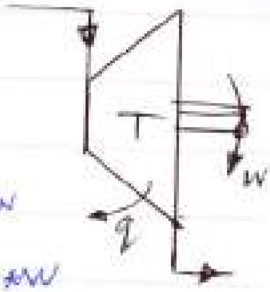
$W = 0$, $\Delta C \simeq 0$, $\Delta Z \simeq 0$

$\therefore q = h_2 - h_1$ kJ/kg heat rejected from a fluid.

$Q = \dot{m}_s (h_2 - h_1) = \dot{m}_w \, C_{pw} (T_{wout} - T_{win})$ " cooling water"

Ex.: In a turbine of gas turbine unit. the gases flow through the turbine at 17 kg/s and the power developed by the turbine is 14000 kw. the enthalpies of the gases at inlet and outlet are 1200 kJ/kg and 360 kJ/kg respectively and the velocities of gas at inlet and outlet are 60 m/s and 150 m/s respectively Find the rate at which heat is rejected from the turbine. Find also the inlet pipe cross-sectional area when the inlet specific volume is 0.5 m³/kg.

Sol.



from Steady flow energy equation.

$g z_1 + h_1 + \frac{1}{2} C_1^2 + q = g z_2 + h_2 + \frac{1}{2} C_2^2 + W$

$z_1 \simeq z_2$, Power = 14000 kw , $P = \dot{m} \times W$

$W = 14000/17 = 823.5$ kJ/kg

$\therefore q = (360 - 1200) + \frac{1}{2}(150^2 - 60^2) \times 10^{-3} + 823.5$

$= -7.05$ kJ/kg $= 7.05 \frac{kJ}{kg}$ rejected heat

$Q = \dot{m} q = 17 \times 7.05 = 119.8$ kw

$\dot{m} = \frac{CA}{v} \Rightarrow A = \frac{17 \times 0.5}{60} = 0.142 \ m^2 = \frac{\pi}{4} D^2 \Rightarrow D = ?$

Specific Heat :-

Is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

$$Q = m \cdot c \cdot \Delta T \qquad kJ$$

c = Specific heat capacity kJ/kg.k
m = mass of substance.
Q = heat transferred to produce temperature change.

Physically the specific heat at constant volume(C_v) can be viewed as the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

The energy required to do the same as the pressure is maintained constant is the specific heat at constant pressure (C_p).

The specific heat at constant pressure (C_p) is always greater than (C_v) because at constant pressure the system is allowed to expand and the energy for thi expansion work must also be supplied to the system.

$$Q = m c \Delta T$$

for unit mass $q = c \cdot \Delta T \Rightarrow dq = c\, dT$

$$\therefore \boxed{c = \frac{dq}{dT}}$$

1. at constant Volume :-

from NFEE

$$q - w = \Delta u \qquad \text{at } v = c \Rightarrow w = 0$$

$$q = \Delta u \Rightarrow dq = du$$

$$\therefore C_v = \frac{dq}{dT} = \frac{du}{dT} \Rightarrow du = C_v \, dT$$

$$\therefore \Delta u = C_v \, \Delta T$$

at Constant Volume $\boxed{Q = \Delta U = m \, C_v \, \Delta T}$

2. at Constant Pressure :-

$$w = P_2 V_2 - P_1 V_1$$

$$q = w + \Delta u = (P_2 V_2 - P_1 V_1) + (U_2 - U_1)$$

$$q = (P_2 V_2 + U_2) - (P_1 V_1 + U_1) = h_2 - h_1 = \Delta h$$

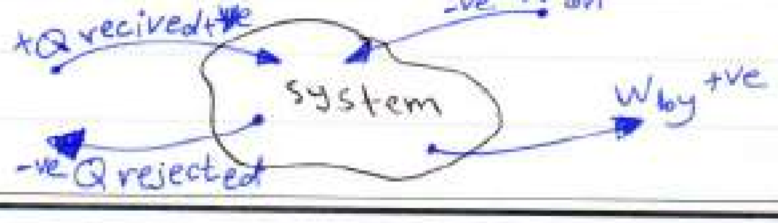
$$dq = dh$$

$$C_p = \frac{dq}{dT} = \frac{dh}{dT} \Rightarrow dh = C_p \cdot dT$$

$$\Delta h = C_p \cdot \Delta T$$

$$\boxed{Q = \Delta H = m \cdot C_p \cdot \Delta T}$$

Note :-



Sheet No.1

Q1. In an air Compressor the Compression take place at Constant internal energy and 50 kJ of heat are rejected to the Cooling water for every kg of air. Find the work required for the compression stroke per kg of air.

Q.2 1 kg of fluid is Compressed reversibly according a $PV = 0.25$. final Volume is one-fourth the initial Volume Calculate the Work done on the fluid and sketch the Process on P-V diagram.

Q.3. $0.05\ m^3$ of gas at 6.9 bar expands reversibly in a cylinder behind a Piston according to law $PV^{1.2} = C$ Until the Volume is $0.08\ m^3$. Calculate the work done by the gas and sketch the Process on P-V dig.

Q4. One kg of fluid expands reversibly according to a linear law from 4.2 bar to 1.4 bar. initial and Volume are $0.004\ m^3$ and $0.02\ m^3$. The fluid is then Cooled reversibly constant Pressure and finally compressed reversibly according to law $PV = c$ back to initial condition of 4.2 bars and $0.04\ m^3$. Calculate the Work done for each Process, the net work done and sketch the cycle on P-v dia

Q.5. In the compression stroke of an internal combustion engine the heat rejected to the cooling water is 45 KJ/kg and the work input is 90 KJ/kg. Calculate the change in internal energy of the working fluid, stating whether it is a gain or a loss.

Q.6. The gases in the cylinder of an internal-combustion engine have an internal energy of 800 KJ/kg and a specific volume of 0.06 m^3/kg at the begining of expansion. The expansion of the gases may be assumed to take place according to a reversible law $pv^{1.5}=c$ from 55 bar to 1.4 bar. The internal energy after expansion is 230 KJ/kg. Calculate the heat rejected to the cylinder cooling water per kg of gases during the expansion stroke.

Q.7. A steady flow of steam enters a condenser with enthalpy of 2300 KJ/kg and a velocity of 350 m/s. The condensate leaves the condenser with an enthalpy of 160 KJ/kg and velocity of 70 m/s. Find the heat transfer to the cooling fluid per kg.

Q.8. A turbine operating under steady flow conditions receives steam at the following state: P=13.8 bars, v=0.143 m^3/kg, u=2590 kJ/kg, V=30 m/s. The state of the steam leaving the turbine is P=0.35 bar, v=4.37 m^3/kg, u=2360 KJ/kg, V=90 m/s. Heat is lost to the surroundings at the rate of 0.25 KJ/s. If the rate of steam flow is 0.38 kg/s. What is the power developed by the turbine.

Q.9. In a Compression Stroke of gas engine the work done on the gas by the Piston is 70 kJ/kg and heat rejected to the cooling water is 42 kJ/kg. find the change of internal energy. stating whether it is gain or lost.

Q10. A nozzle is a device for increasing the velocity of fluid. At inlet to the nozzle the enthalpy is 3026 kJ/kg and the velocity is 60 m/s. At exit from the nozzle the enthalpy is 2790 kJ/kg. The nozzle is horizontal and there is negligible heat loss from the turbine.

a- find the velocity at the nozzle exit.
b- If the inlet area is 0.1 m^2 and the specific volume at the inlet is 0.19 m^3/kg. find the mass flow rate.
c- If $v = 0.5$ m^3/kg find the exit area of the nozzle.

CHAPTER: TWO

IDEAL GAS

* ~~~ * ~~~ * ~~~ *

- Ideal gas :- Boyle's Law , Charles's Law

- The characteristic equation of a perfect gas.
- Relation Between Specific Heat C_p , C_v
- Reversible and Irreversible Processes for the ideal gas.

1. Reversible non-flow Processes

a- Constant volume process (Isometric)

b- " Pressure " (Isobaric)

c- " temperature " (Isothermal)

d- Adiabatic process, the Relation between P & V and T

e- Polytropic "

2. Irreversible Process (closed System)

a- Paddle work (P=c) . b- Paddle work (V=c) .

3. Reversible Process (open System)

a- Constant pressure process.

b- Constant temperature. C- Polytropic.

4. Irreversible process (open system)

a- Unresisted or free expansion.

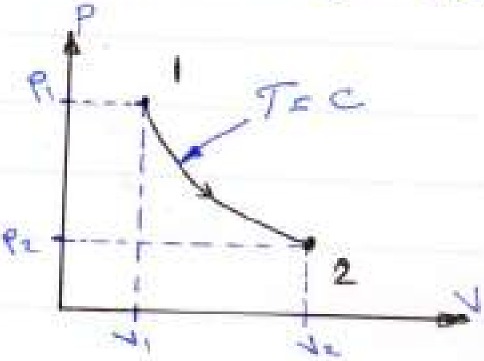
b- Throttling Process.

c- Adiabatic mixing.

IDEAL GAS :- (Perfect gas)

1. Boyle's Law :-

It is defined as : when a change of state of any gas in which the mass and the temperature remains Constant, the volume varies inversely as the Pressure.



$$P \propto \frac{1}{V} \Rightarrow P = C * \frac{1}{V}$$

$$\therefore PV = \text{Constant} \quad , \quad T = \text{Const.}$$

$$\boxed{PV = C}$$ ---- Boyle's Law.

Ex. :-

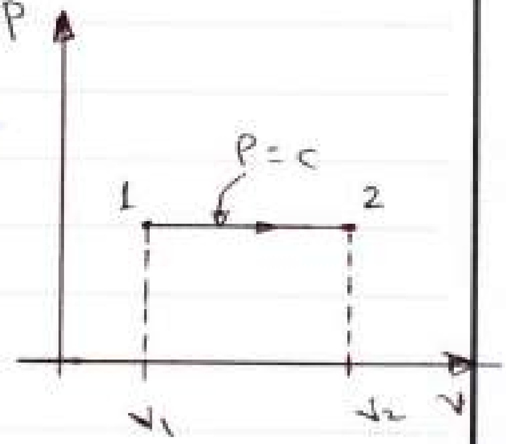
Agas whose original Pressure and Volume were 300 kpa. and 0.14 m^3 is expanded until it's new Pressure is 60 kpa. while its temperature remains Constant. What is its new volume.

Sol. : $P_1V_1 = P_2V_2 \Rightarrow V_2 = \frac{P_1}{P_2} * V_1$

$$V_2 = 0.14 * \frac{300}{60} = 0.7 m^3$$

2. Charles's Law :-

It is defined as: when the change of state of any gas in which the mass and pressure remain constant, the volume varies in proportion with the absolute temperature.



$$V \propto T \Rightarrow V = C * T$$

$$\therefore \boxed{\frac{V}{T} = C}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad , P = \text{Constant}.$$

Ex.: A quantity of gas whose original volume and temp. are 0.2 m³ and 303 °c respectively, is cooled at constant pressure until its volume becomes 0.1 m³. what will be the final temperature of the gas.

Sol. :-

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

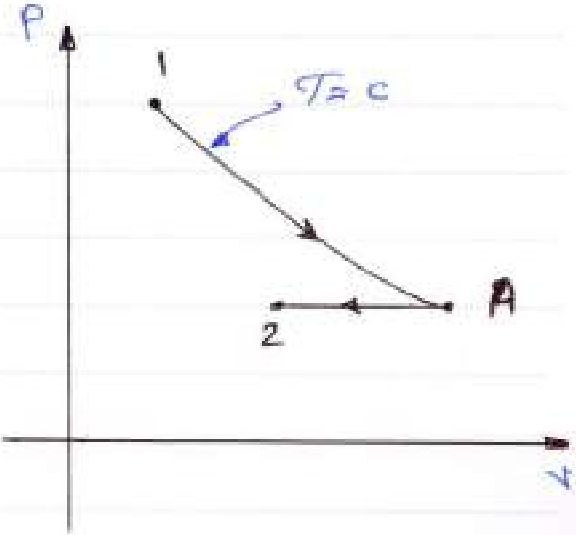
$$\Rightarrow T_2 = T_1 * \frac{V_2}{V_1}$$

$$T_1 = 303 + 273 = 576 \; k$$

$$\therefore T_2 = 576 * \frac{0.1}{0.2} = 288 \; k$$

$$t_2 = 288 - 273 = 15 \; °c$$

The characteristic equation of a perfect gas :-



the Process $1 \rightarrow A$

$PV = C$ Boyles , $T = \text{Const.}$

$\therefore P_1V_1 = P_AV_A$, $T = C \Rightarrow T_1 = T_A$

$V_A = \frac{P_1}{P_A} * V_1$ ---- (1)

the Process $A \rightarrow 2$

$V = C * T$, Charles , $P = \text{Const.}$

$\frac{V_A}{T_A} = \frac{V_2}{T_2} = \text{Const.}$, $P_A = P_2$

$V_A = \frac{T_A}{T_2} * V_2$ ---- (2)

from Boyle's Law $T_A = T_1$ Sub. in eq. (2)

$V_A = \frac{T_1}{T_2} * V_2$ ------ (3)

from eq. (1) & (3) $\Rightarrow \frac{T_1}{T_2} V_2 = \frac{P_1}{P_A} * V_1$; $P_A = P_2$

$\therefore \frac{P_1}{P_2} V_1 = \frac{T_1}{T_2} V_2 \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{Constant}$

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = R}$$ R = gas Constant

$P_1 V_1 = R T_1$ for 1 kg
$P_1 V_1 = m R T_1$ for m kg
$m = n \cdot M$
where : m = mass kg
n = number of moles mol
M = molecular weight

$PV = mRT$ In general.
$PV = n \cdot MRT \Rightarrow MR = \frac{PV}{n \cdot T}$ (P & T are Const.)

$MR = C * \frac{V}{n}$; $\frac{V}{n} = c$ (Avagador's theorm)

$\therefore MR = R_o$; R_o: Universal gas Constant.

Avogadros theorm : Volume of one mole of any gas is same for all gases at any temperature and Pressure.

$\therefore R_o = \frac{Pv}{nT}$

$\because$ Volume of 1 mole at 0 °C and $P = P_{atm} = 1.01325$ bars equal to 22.7 m^3

$$\therefore \; R_o = \frac{1.01325 \times 10^5 \times 22.7}{1 \times (0+273)} = 8.314 \text{ kJ/kg.K}$$

for O_2 : $M=32 \Rightarrow R = \frac{R_o}{M} = \frac{8314}{32} = 259 \frac{J}{kgK}$

Ex. :- A Volume of $3.6 m^3$ of O_2 initially at 220°C and Pressure 400 kPa, is Compressed reversibly at Constant temperature to final volume of $0.06 m^3$. Calculate the mass - the final Pressure, the increase in internal energy, and the work done.

Sol. :-

for O_2, $M=32 \Rightarrow R = \frac{8.314}{32} = 0.259$ kJ/kgK

$$PV = mRT \Rightarrow m = \frac{400 \times 10^3 \times 3.6}{259 \times (220+273)} = 11.23 \text{ kg}$$

$$P_1V_1 = P_2V_2 \,,\; T=c \Rightarrow P_2 = \left(\frac{3.6}{0.06}\right) \times 400 = 24 \text{ MPa}$$

from NFEE $Q = W + \Delta U$, $\Delta U = 0 \Rightarrow T = c$

$$W = \int_1^2 P dv \;;\; PV = C \Rightarrow P = \frac{C}{V}$$

$$\therefore W = \int_{v_1}^{v_2} \frac{C}{V} dV = C\int_{v_1}^{v_2} \frac{dV}{V} = C \times (\ln V_2 - \ln V_1)$$

$$W = P_1V_1 \times \ln \frac{V_2}{V_1} = 400 \times 3.6 \times 10^3 \times \ln \frac{0.06}{3.6} = -5.896 \text{ MJ}$$

$$Q = W = -5.896 \text{ MJ}$$

Joule's Law :-

It is defined as :-

The internal energy of a gas is a function of temperature only and is independent of changes in pressure and Volume.

$\therefore$ If $T = C \Rightarrow \Delta U = 0$

- The specific heat capacities of a gas :-

a- the Constant Volume heating of a gas :

Let a mass of a gas be heated at constant Volume such that its temperature rises from T_1 to T_2 and its pressure rises from P_1 to P_2. Then

Heat received by the gas $= m\, C_v\, (T_2 - T_1)$

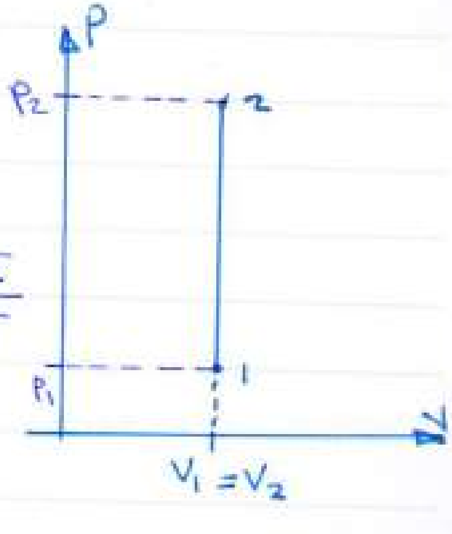
from NFEE, $Q = \Delta U + W$

$\therefore V = C \Longrightarrow W = 0 \Rightarrow Q = \Delta U$

or $m C_v (T_2 - T_1) = U_2 - U_1$

from the Perfect gas eq.

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \Rightarrow P_2 = P_1 \times \frac{T_2}{T_1}$$



Ex.:- 2 kg of a gas occupying 0.7 m^3, had an original temperature of 15 °C. It was then heated at constant volume until its temperature became 135 °C. How much heat was transferred to the gas and what was its final pressure? Take, $C_v = 0.72$ kJ/kgK and $R = 0.29$ kJ/kgK.

Sol.:- V = Constant

$$Q = W + \Delta U, \quad W = 0 \quad V = C$$

$$Q = \Delta U = m C_v (T_2 - T_1)$$

$$Q = 2 * 0.72 * (135 - 15) = 172.8 \text{ kJ}$$

$$P_1 V_1 = m R T_1$$

$$P_1 = \frac{2 * 0.29 * 288}{0.7} = 238.6 \text{ kPa}$$

Since the volume remains constant, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = P_1 * (T_2 / T_1) = 238.6 * \frac{408}{288}$$

$$\therefore P_2 = 338.1 \text{ kPa}.$$

or $$P_2 V_2 = m R T_2 \Rightarrow P_2 = \frac{m R T_2}{V_2}$$

$$V_1 = V_2$$

$$P_2 = \frac{2 * 0.29 * (135 + 273)}{0.7} = 338.1 \text{ kPa}.$$

b. the constant pressure heating of a gas :-

Let a mass of gas be heated at constant pressure such that its temperature rises from T_1 to T_2 and its volume increases from V_1 to V_2 Then.

Heat received by the gas $= m C_p (T_2 - T_1)$

$$Q = W + \Delta U$$

$$m C_p (T_2 - T_1) = P(V_2 - V_1) + U_2 - U_1$$

$$= (PV_2 + U_2) - (PV_1 + U_1) = H_2 - H_1$$

from the characteristic eq. of perfect gas:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad , P_1 = P_2 \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Note : $H = m C_p T$, $h = C_p T$, $\Delta h = C_p \Delta T$

$U = m C_v T$, $u = C_v T$, $\Delta u = C_v \Delta T$

Ex.: A gas whose pressure, volume and temp. are 275 kPa, 0.09 m^3 and 185 °C, respectively, has its state changed at constant pressure until its temp. becomes 15 °C. How much heat is transferred from the gas and how much work is done on the gas during the process? $R = 0.29$ kJ/kgK , $C_p = 1.005$ kJ/kgK

Sol. $P_1 V_1 = m R T_1 \Rightarrow m = 275 \times 10^3 \times 0.09 / (0.29 \times 10^3 \times 458)$

$m = 0.186$ kg.

$Q = m C_p (T_2 - T_1) = 0.186 \times 1.005 (288 - 458) = -31.78$ kJ

(the heat has been extracted from the gas)

$P = c \Rightarrow V_2 = V_1 \times \frac{T_2}{T_1} = 0.09 \times \frac{288}{458} = 0.0566 \ m^3$

$\therefore W = P(V_2 - V_1) = 275 \times (0.0566 - 0.09) = -9.19$ kJ

- Relation Between Specific Heat "C_p, C_v" :-

from the non-flow energy eq. NFEE

$$Q = W + \Delta U$$

$$W = \int_1^2 P \cdot dV = P_2V_2 - P_1V_1$$

Ideal gas law $PV = mRT$

$$\therefore W = mR(T_2 - T_1)$$

$$\therefore Q = mR(T_2 - T_1) + mC_v(T_2 - T_1)$$

$$= m(R + C_v)(T_2 - T_1)$$

$$\therefore Q = \Delta H = mC_p(T_2 - T_1)$$

$$\therefore mC_p(T_2 - T_1) = m(R + C_v)(T_2 - T_1)$$

$$\therefore C_p = R + C_v$$

$$\Rightarrow \boxed{(C_p - C_v) = R}$$

$\gamma = \frac{C_p}{C_v}$, γ = the ratio of the specific heat.

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v} \Rightarrow \gamma - 1 = \frac{R}{C_v}$$

$$\therefore \boxed{C_v = \frac{R}{\gamma - 1}}$$

$$C_p = \gamma * C_v \Longrightarrow \boxed{C_p = \frac{\gamma R}{\gamma - 1}}$$

Ex.: A perfect gas have specific heat as $C_p = 0.846 \frac{kJ}{kgK}$ $C_v = 0.657$ kJ/kgK. find the gas constant and molecular weight of gas.

Sol.: $R = C_p - C_v = 0.846 - 0.657 = 0.189$ kJ/kgK

$M = R_o/R = (8.314/0.189) = 44$

Ex.: A perfect gas has molecular weight of 26 and a value of $\gamma = 1.26$, Calculate the heat rejected for 1 kg when a- the gas is contained in a rigid vessel at 3 bars and 315°C and cooled until the pressure fails to 1.5 bars. b- the gas enters pipe at 280°C and flow steady in the end of the pipe where T= 250°C. Neglect any chang in velocity of the gas.

Sol.: a) $Q = W + \Delta U$, rigid vessel $\Rightarrow V = c \Rightarrow dv = 0$

$\because W = \int p\,dv \Rightarrow W = 0$

$\therefore Q = \Delta U = m C_v (T_2 - T_1)$

$M = 26, \gamma = 1.26$, $R = R_o/M = (8.314/26) = 0.3198 \frac{kJ}{kgK}$

$C_v = \frac{R}{\gamma - 1} = \frac{0.3198}{1.26 - 1} = 1.229$ kJ/kgK

$\therefore Q = 1 \times 1.226 \times (T_2 - T_1)$.

Since $v = c$ $\Rightarrow$ from the characteristic eq.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow T_2 = \frac{P_2}{P_1} \times T_1 = \frac{1.5}{3}(315 + 273)$$

$\therefore T_2 = 294\ K \Rightarrow Q = 1 \times 1.266 (294 - 588) = -361$ kJ

$= 361$ kJ rejected

b) Steady flow Energy Equation.

$P_1V_1 + U_1 + \frac{1}{2}C_1^2 + gZ_1 + Q = P_2V_2 + U_2 + \frac{1}{2}C_2^2 + gZ_2 + W$

$\Delta Z \simeq 0$, $\Delta C \simeq 0$, $W = 0$

$\therefore q = h_2 - h_1$

$Q = m(h_2 - h_1) = mC_p(T_2 - T_1)$; $C_p = \gamma * C_v$

$C_p = 1.26 * 1.229 = 1.548$ kJ/kg K.

$Q = 1 * 1.548 * (250 - 280) = -46.44$ kJ

$= 46.44$ kJ rejected.

Ex.:- Five kilogram of oxygen are heated from 250K to 400K at constant pressure. Calculate the change of enthalpy, change in internal energy, heat transferred and work done. $\gamma = 1.4$

Sol.:- a non-flow energy eq.

$Q = W + \Delta U$; $W = \int P dv = P(V_2 - V_1)$, $P = C$ given.

$Q = (P_2V_2 - P_1V_1) + (U_2 - U_1) = (P_2V_2 + U_2) - (P_1V_1 + U_1)$

$Q = H_2 - H_1 = mC_p(T_2 - T_1)$ for $P = C$ only.

for O_2 $M = 32$, $\gamma = 1.4$, $R = R_0/M = (8.314/32) = 0.2598 \frac{kJ}{kg\,K}$

$C_p = \frac{\gamma R}{\gamma - 1} = (1.4 * 0.2598/0.4) = 0.9093$ kJ/kg.K

$\Delta H = 5 * 0.9093 * (400 - 250) = 692$ kJ

$\Delta U = mC_v(T_2 - T_1)$; $C_v = \frac{R}{\gamma - 1} = (0.2598/0.4) = 0.6495 \frac{kJ}{kg\,K}$

$\Delta U = 5 * 0.6495 * (400 - 250) = 487.125$ kJ

$Q = W + \Delta U = \Delta H$ at $P = Const.$

$\therefore W = Q - \Delta U = 692 - 487.125 = 204.875$ kJ

work done by the system.

Ex.: An Oxygen Cylinder has a capacity of 300L, and Contains O_2 at 3.1 MPa and 18 °C. The Valve is open and Some gas is used. If the pressure and temp. of the Oxygen fall to 1.7 MPa and 15 °C. Find the mass of oxygen used. If after the valve is closed the oxygen remain in the Cylinder is gradualy attains its initial temp. of 18 °C. Find the amount of heat transfer through the Cylinder wall, Oxygen density at 0 °C and 0.101325 MPa is 1.429 kg/m³.

Sol.:- $P_1 V_1 = m R T_1 \Rightarrow m = \frac{P_1 V_1}{R T_1}$

$$R = (8.314/32) = 0.26 \ \text{kJ/kg K}$$

∴ initial mass $= (3.1\times10^{3} \times 300\times10^{-3})/(0.26\times(18+273)) = 12.29$ kg

∴ $m_1 = 12.29$ kg

final mass $= (1.7\times10^{3}\times300\times10^{-3})/(0.26\times(15+273)) = 6.8$ kg

∴ $m_2 = 6.8$ kg

mass used $= m_1 - m_2 = 12.29 - 6.8 = 5.49$ kg

$Q = W + \Delta U$, $V = c \Rightarrow W = 0 \Rightarrow Q = \Delta U$

∴ $Q = m C_v (T_2 - T_1)$; $C_v = (R/(\gamma - 1)) = (0.26/0.4) = 0.65$ kJ/kg K

∴ $Q = 6.8 \times 0.65 (18 - 15) = 13.26$ kJ

Sheet 2.

Q1. The molecular weight of carbon dioxide (CO_2) is 44. In an experiment the value of γ was found to be 1.3. Assuming that CO_2 is a perfect gas. Calculate the gas constant R and the specific heats at constant pressure and constant volume. Ans.[0.189, 0.63, 0.819 $\frac{kJ}{kg\,K}$]

Q2. Calculat the internal energy and enthalpy of 1 kg of air occupying 0.05 m^3 at 20 bars. If the internal energy is increased by 120 kJ/kg as the air compressed to 50 bars. Calculate the new volume occupied. Ans.[250, 350 $\frac{kJ}{kg\,K}$, 0.0296 m^3]

Q3. When a certain perfect gas is heated at constant pressure from 15°c to 95°c, the heat required 1130 kJ/kg. While when the same gas is heated at constant volume between the same temperature limits above, the required is 808 kJ/kg. Calculate C_p, C_v, γ, R and M of the gas.

Q4. In an air compressor the pressure at inlet and outlet are 1 bars & 15 bars respectively. The temperature of the air at inlet is 15°c and the volume at beging of compression is three times that at the end of the compression. Calculate the temp. at outlet and the increase in internal energy.

Q5. The exhaust gas leaving an oil engine is passed through a heat exchanger which consist of tubes surrounding by water. The gas enters at 327°C and leaves at 193°C. The water enters the exchanger at 17°C and leaves at 56°C. If the mass flow rate of water is 2.54 kg/min Determine the mass flow rate of gas.

$C_p = 1.04$ kJ/kgK , $C_{pw} = 4.186$ kJ/kgK

Ans.[2.98 kg/min]

Q6.

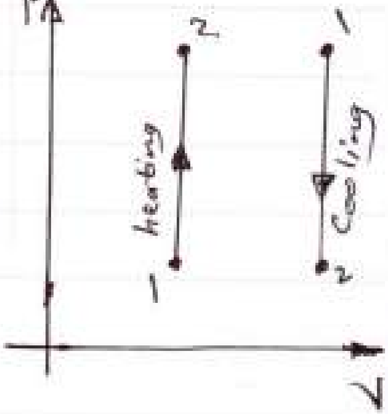
A working fluid enters a steady-flow system with velocity 30 m/s and leaves with velocity 140 m/s. The mass flow rate is 9 kg/s. The Properties of the fluid are at entry, 13.8 bars, 0.122 m^3/kg, internal energy 422 kJ/kg and exit properties are 1.035 bars, 0.805 m^3/kg, internal energy 208 kJ/kg. The heat transfer from the System is 4.22 kJ/kg. Determine the work transfered in kw from the System.

Reversible and Irreversible Processes for The Ideal Gas.

1. Reversible non-flow Processes:

a- Constant Volume Process (Isometric).
from the non flow E.E

$Q = W + \Delta U$



$W = \int p\,dv \quad , V = c \Rightarrow dv = 0$

$\therefore W = 0$

$\Rightarrow \quad Q = \Delta U$

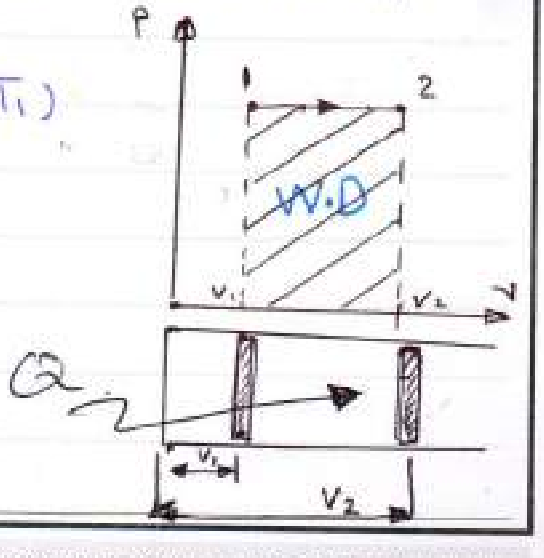
$$\boxed{Q = m C_v (T_2 - T_1)}$$

b- Constant Pressure Process (Isobaric)
from the non-flow E.E.

$Q = W + \Delta U \;;\; W = P_2 V_2 - P_1 V_1 = mR(T_2 - T_1)$

$\Delta U = m C_v (T_2 - T_1)$

$$\begin{aligned} \therefore \; Q &= mR(T_2 - T_1) + mC_v(T_2 - T_1) \\ &= m(R + C_v)(T_2 - T_1) \\ &= m C_p (T_2 - T_1) = \Delta H \end{aligned}$$



$Q = m C_p (T_2 - T_1)$

Ex. : 0.05 kg of air is heated at constant Pressure of 2 bars until the volume occupied is 0.0658 m³. Calculate the heat supplied and work done. when the initial temperature is 130 °C .

Sol. :-

the non-flow energy eq.

$$Q = W + \Delta U$$

$$W = P(V_2 - V_1) = mR(T_2 - T_1)$$

$$\Delta U = mC_v(T_2 - T_1)$$

$$\therefore Q = mC_p(T_2 - T_1)$$

$$P_2 V_2 = mRT_2 \Rightarrow T_2 = (2 * 10^2 * 0.0658)/(0.05 * 0.287)$$

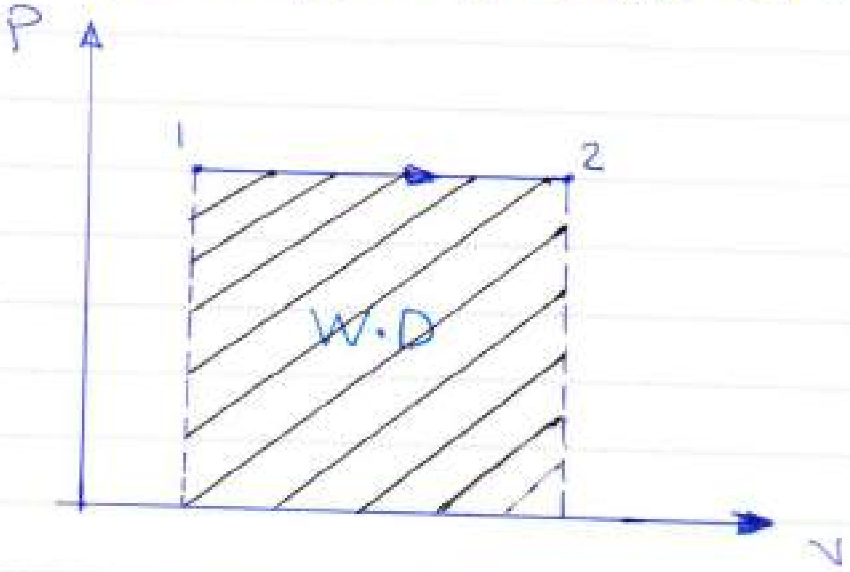
$$\therefore T_2 = 912 \, k$$

$$\therefore Q = 0.05 * 1.005 (912 - 403) = 25.6 \, kJ$$

$$W = 0.05 * 0.287 * (912 - 403) = 7.38 \, kJ$$

$$\Delta U = 0.05 * 0.717 * (912 - 403) = 18.27 \, kJ$$

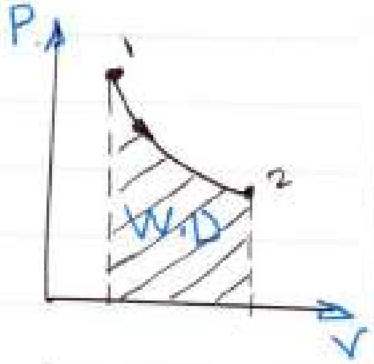
$$Q = W + \Delta U = 7.38 + 18.27 = 25.65 \, kJ.$$



C. Constant temperature Process (Isothermal) from non-flow E.E

$Q = W + \Delta U$; $\Delta U = m C_v (T_2 - T_1)$

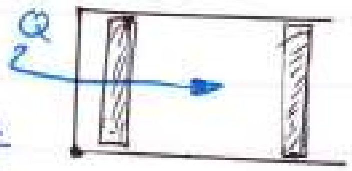
Since $T = C \Rightarrow \Delta U = 0$



$W = \int P\,dV$, we have $\frac{PV}{T} = C$

$T =$ Const. $\Rightarrow PV = C \Rightarrow P = \frac{C}{V}$

$W = \int \frac{C}{V}\,dV = C * \ln\frac{V_2}{V_1}$



$\therefore W = PV\ln\frac{V_2}{V_1} = mRT_1\ln\frac{V_2}{V_1}$

$$\therefore \boxed{Q = PV\ln\frac{V_2}{V_1} = PV\ln\frac{P_1}{P_2} = mRT\ln\frac{P_1}{P_2}}$$

Ex. A quantity of gas has initial Pressure 140 kpa and Volume 0.14 m^3. It is than compressed as a pressure of 700 kpa while the temperature remains Constant. Determine the final volume of the gas.

Sol. $T = C$

from characteristics eq. $\Rightarrow \frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}}$

$T_1 = T_2 \Rightarrow P_1V_1 = P_2V_2$

$\Rightarrow V_2 = \frac{P_1}{P_2} * V_1 = \frac{140}{700} * 0.14 = 0.028\ m^3$

Ex. 1 kg of nitrogen Molecular weight 28 is compressed reversibly and Isothermally from 1.01 bar & 20 °C to 4.2 bar Calculate the work done and the heat flow during the Process. Assuming nitrogen to be a perfect gas.

Sol.

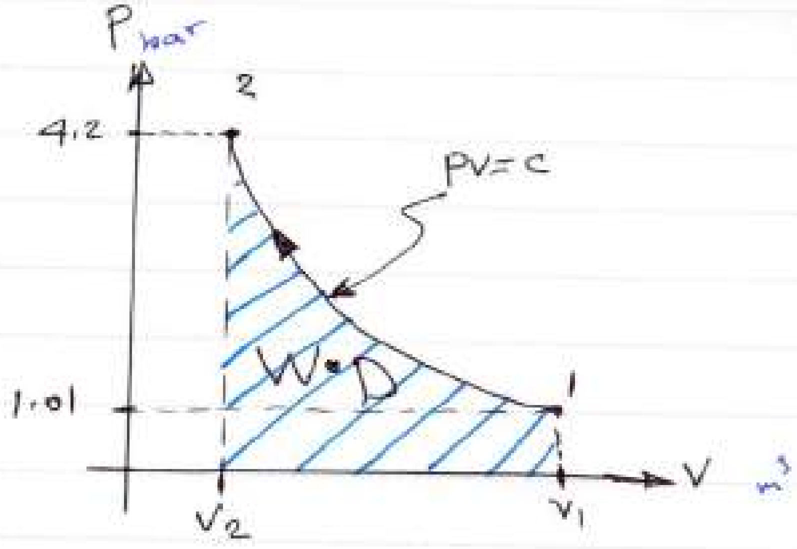
$$R = \frac{R_0}{M} = 8314/28 = 0.297 \text{ kJ/kg K}$$

$$W = mRT \ln \frac{P_1}{P_2} = 0.297 * 293 \cdot \ln \frac{1.01}{4.2} = 124 \frac{kJ}{kg}$$

$$\therefore Q = W \Rightarrow \therefore Q = -124 \frac{kJ}{kg}$$

$$W = +124 \frac{kJ}{kg} \text{ (Comp.)}$$

$$Q = +124 \text{ kJ/kg (heat rejected.)}$$



d. Adiabatic Process:

In this Process the System must be in thermal insulated to prevent the heat transfer between the System and surrounding.

To find the Relation between P & V from non-flow E.E.

$Q = W + \Delta U$, adiabatic Process $\Rightarrow Q = 0$

$$\left.\begin{array}{lll} dQ = dW + dU & \Rightarrow & dq = dw + du \\ dU = mc_v\, dT & \Rightarrow & du = c_v\, dT \\ dW = P\, dV & \Rightarrow & dw = P\, dv \end{array}\right] \text{Per unit mass}$$

$$\therefore dq = c_v\, dT + P\, dv$$

$$PV = RT \Rightarrow P = \frac{RT}{v}$$

$$\therefore \left(dq = c_v\, dT + RT\frac{dv}{v}\right) \div T$$

$$0 = c_v \frac{dT}{T} + R\frac{dv}{v} \quad \div c_v$$

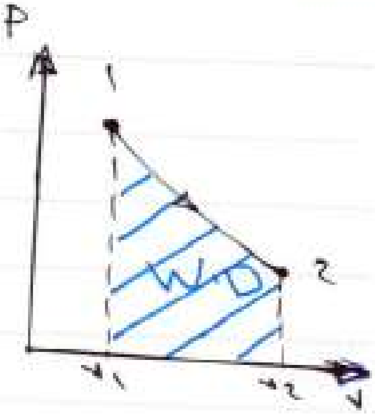
$$0 = \frac{dT}{T} + \frac{R}{c_v}\frac{dv}{v} \quad ; \quad c_v = \frac{R}{\gamma - 1}$$

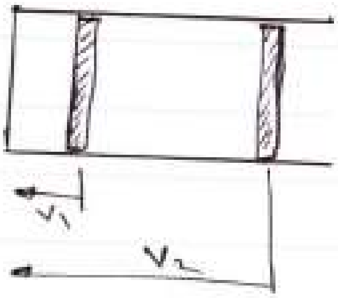
$$\Rightarrow \therefore \frac{R}{c_v} = (\gamma - 1)$$

$$0 = \frac{dT}{T} + (\gamma-1)\frac{dv}{v}$$

$$\int 0 = \int_{T_1}^{T_2} \frac{dT}{T} + (\gamma-1)\int_{v_1}^{v_2} \frac{dv}{v}$$

$$\ln T + (\gamma-1)\ln V = c$$





$$PV = RT \Rightarrow T = \frac{PV}{R}$$

$$\ln\left(\frac{PV}{R}\right) + (\gamma-1)\ln V = c$$

$$\ln \frac{PV\cdot V^{(\gamma-1)}}{R} = c$$

$$\frac{PV^{\gamma}}{R} = e^{c} \Rightarrow PV^{\gamma} = R\times e^{c} = c$$

$$\therefore PV^{\gamma} = c \quad \text{adiabatic process}$$

The non-flow E.E.

$$Q = W + \Delta U \; ; \; Q = 0 \Rightarrow W = -\Delta U$$

$$W = m c_v (T_1 - T_2) = m\frac{R}{\gamma-1}(T_1 - T_2)$$

$$\therefore \boxed{W = \frac{mR(T_1 - T_2)}{(\gamma-1)} = \frac{P_1V_1 - P_2V_2}{(\gamma-1)}}$$

Adiabatic work

– The Relation between P & V and T in Adiabatic Process :-

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1} * \frac{T_1}{T_2} \text{ ----- (1)}$$

adiabatic Process $\Rightarrow P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \text{ ------ (2)}$$

Sub. eq. (1) in eq. (2)

$$\left(\frac{V_2}{V_1}\right)^\gamma = \left(\frac{V_2}{V_1}\right) * \left(\frac{T_1}{T_2}\right)$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \text{ ------ (3)}$$

Sub. eq. (2) in eq. (3)

where $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$

$$\frac{T_1}{T_2} = \left[\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}\right]^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \boxed{\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}}$$

Ex. Air at 1.02 bar, 22°C initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bars. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder.

Sol.

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 * \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = (22+273) * \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} = 507.5\ K$$

$$T_2 = \text{final temp.} = 507.5\ K$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} * V_1$$

$$\therefore V_2 = 0.015 * \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} = 0.00388\ m^3$$

$Q = W + \Delta U$, $Q = 0$ adiabatic pro.

$$\therefore W = U_1 - U_2 = m\, C_v (T_1 - T_2)$$

$$P_1 V_1 = m R T_1 \Rightarrow m = P_1 V_1 / R T_1$$

$$m = (1.02 \times 10^5 \times 0.015) / (0.287 \times 10^3 \times 295)$$

$$m = 0.0181\ kg$$

$$\therefore W = 0.0181 * 0.717 * (295 - 507.5) = -2.76\ kJ$$

total work input = 2.76 kJ

e- Polytropic Process :-

This general case of expansion or Compression of a Substance according to the law $PV^n = c$, It is Called a Polytropic expansion or compression or a Polytrope.

It is should be noted that the Value of the Constant "**c**" will change with each change of condition, So also will the value of "n" which is called the "index" of the expansion or compression or the Polytropic exponent.

$$PV^n = c$$

The non-flow energy equation

$$Q = W + \Delta U$$

$$W = \int P\,dv = c\int \frac{dv}{v^n} = c\int V^{-n}\,dV$$

$$W = PV^n\left[\frac{V^{-n+1}}{-n+1}\right]_1^2 = \frac{P_2V_2^n \cdot V_2^{-n+1} - P_1V_1^n \cdot V_1^{-n+1}}{-n+1}$$

$$W = \frac{P_2V_2 - P_1V_1}{-n+1} = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$W = \frac{mR(T_1 - T_2)}{n-1}$$

$$\Delta U = m c_v (T_2 - T_1) = - m c_v (T_1 - T_2)$$

$$c_v = (R / \gamma - 1)$$

$$\Delta U = \frac{-mR(T_1 - T_2)}{\gamma - 1}$$

$$\therefore Q = \frac{mR(T_1 - T_2)}{n-1} - \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= mR(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma - 1}\right)$$

$$= mR(T_1 - T_2)\left(\frac{\gamma - \cancel{1} - n + \cancel{1}}{(n-1)(\gamma - 1)}\right)$$

$$= mR(T_1 - T_2)\left(\frac{\gamma - n}{(n-1)(\gamma - 1)}\right)$$

$$= \frac{mR(T_1 - T_2)}{n-1} * \left(\frac{\gamma - n}{\gamma - 1}\right)$$

$$\boxed{\therefore Q = \left(\frac{\gamma - n}{\gamma - 1}\right) * W}$$

In a polytropic Process the value of n depends on the a amount of heat added or rejected and on the work done during the Process $PV^n = C$

when :

1. $n = 0 \longrightarrow PV^0 = c \longrightarrow P = c$ (Isobaric)
2. $n = 1 \longrightarrow PV^1 = c \longrightarrow PV = c$ (Isothermal)
3. $n = \gamma \longrightarrow PV^\gamma = c \longrightarrow Q = 0$ (adiabatic)
4. $n = \infty \longrightarrow PV^\infty = c \rightarrow P^{1/\infty} V = c \longrightarrow V = c$ (Isometric) or (Isochoric)

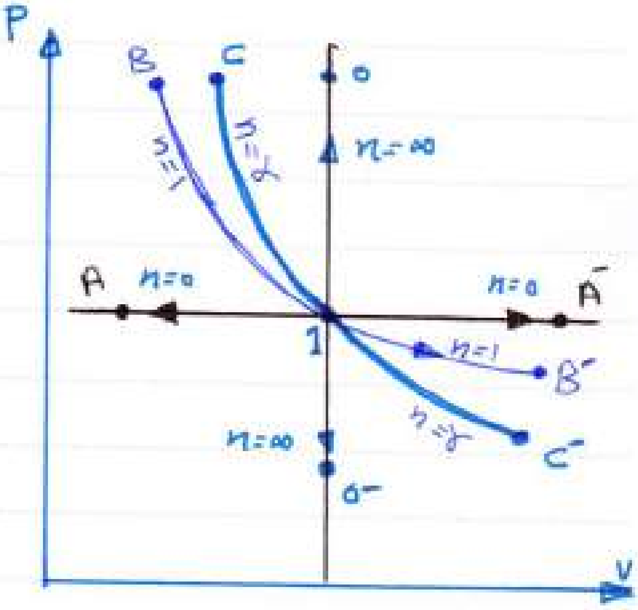
Processes:-

$1 \rightarrow A$ Cooling at $P=c$
$1 \rightarrow A'$ Heating at $P=c$

$1 \rightarrow B'$ Expansion at $T=c$
$1 \rightarrow B$ Compression at $T=c$

$1 \rightarrow C$ Adiabatic Comp. $PV^\gamma = c$
$1 \rightarrow C'$ 〃 Expansion

$1 \rightarrow D$ Heating at $v=c$
$1 \rightarrow D'$ Cooling 〃 〃



- Work and the hyperbolic Process :-

The hyperbolic Process is a Particular Case of the Polytropic Process, $PV^n = C$, being the Case when $n=1$. Thus, the law for the hyperbolic Process is $PV = C$

For a hyperbolic change from State 1 to State 2.

$$P_1V_1 = P_2V_2$$

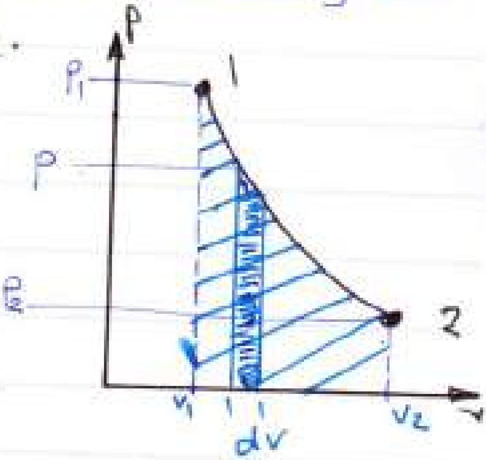
An expression for the work done during a Polytropic Process has already been determined.

$$\text{Work done} = \frac{P_1V_1 - P_2V_2}{n-1}$$

for hyperbolic process. $P_1V_1 = P_2V_2$ & $n=1$

$$\therefore \text{Work done} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{0}{0}$$

to compute the work done. It is necessary to determine the area under the curve.



$$\text{Work done} = \int_{V_1}^{V_2} P\,dV \quad , P = \frac{C}{V}$$

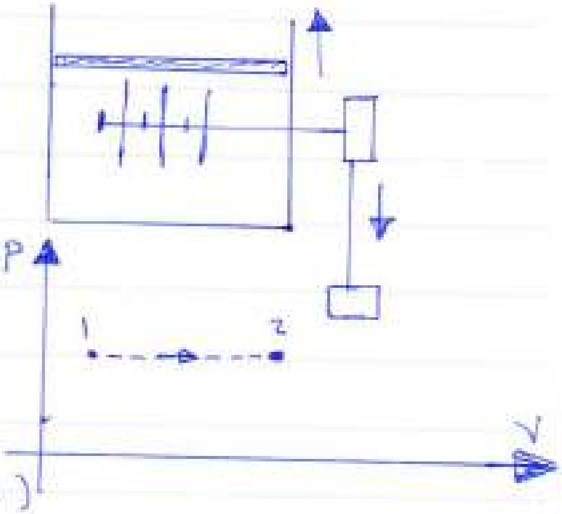
$$W = C\int_{V_1}^{V_2} \frac{dV}{V} = C\left[\ln V\right]_{V_1}^{V_2}$$

$$= C\left[\ln V_2 - \ln V_1\right] = C\ln\frac{V_2}{V_1}$$

$$\boxed{W = PV\ln\frac{V_2}{V_1}}$$

2. Irreversible Processes (close system)

a. Paddle work ($P = C$)



W_p = Paddle work

from non-flow E.E

$Q = \Delta U + W + W_p$

$\Delta U = m C_v (T_2 - T_1)$

$W = P(V_2 - V_1) = mR(T_2 - T_1)$

$W_p = -ve$ since it is done on the system

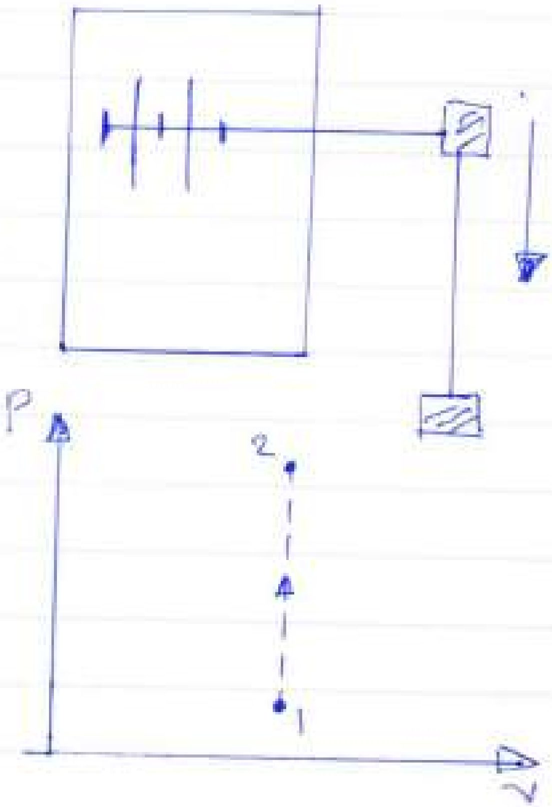
b. Paddle work $V = c$

$Q = W + \Delta U + W_p$

$W = 0$ $v = c$, $Q = 0$

$\Delta U = m C_v (T_2 - T_1)$

$W_p = -ve$ done on the system.



Ex.:- A constant pressure adiabatic system contain 0.13 kg of air at 1.3 bar. The system recives paddle work. The temperature of air rises from 29 to 185 °C. Find the total work. mechanical work, change in internal energy and enthalpy. Take $R = 0.287\ kJ/kg.K$, $\gamma = 1.4$

Sol.:- $Q = W + \Delta U + W_p$, $Q = 0$ adiabatic

$W = mR(T_2 - T_1) = 0.13 * 0.287(185 - 29) = 5.82\ kJ$

$\Delta U = mC_v(T_2 - T_1)$

$\Delta U = 0.13 * 0.717(185 - 29) = 14.5\ kJ$

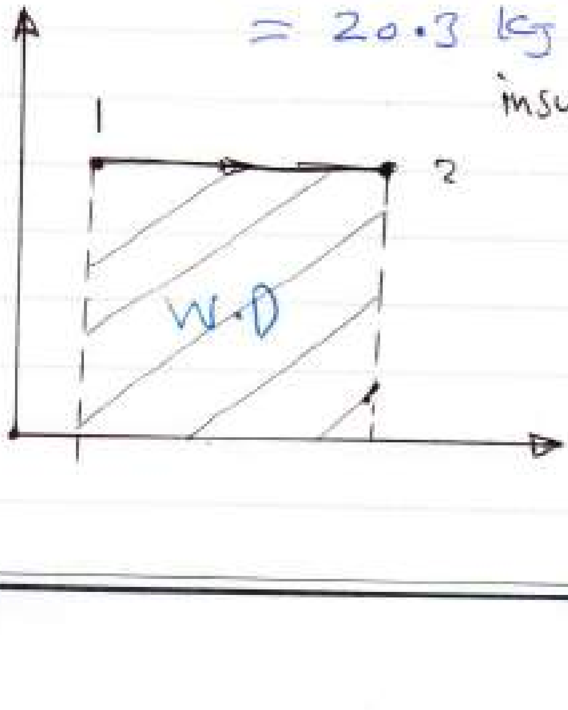
$Q = W + \Delta U + W_p$

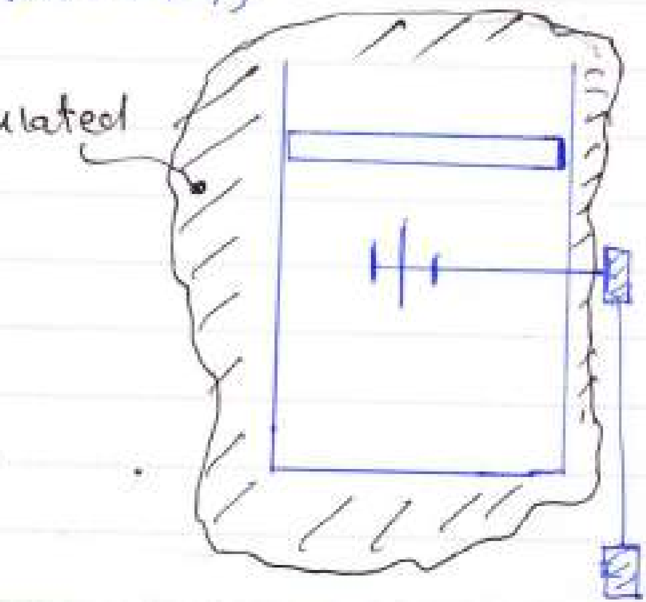
$0 = 5.82 + 14.5 + W_p \Rightarrow W_p = -20.32\ kJ$

$\Delta H = mC_p(T_2 - T_1)$

$= 0.13 * 1.005(185 - 29)$

$= 20.3\ kJ$





Sheet No. 3

Q.1

Oxygen ($M = 32$) expands reversibly in a cylinder behind a piston at constant pressure of 3 bars. the initial volume is 0.01 m^3 and finally is 0.03 m^3. the initial temperature is 17 °C. Calculate the work done by the Oxygen and the heat flow to or from the cylinder walls during the expansion. Take $C_p = 0.917$ kJ/kgK. Ans: [6 kJ , 21.25 kJ]

Q2

0.05 m^3 of a perfect gas at 6.3 bars undergoes a reversible isothermal process to a pressure of 1.05 bars. Calculate the heat flow to or from the gas. Ans: [56.4 kJ]

Q3

1 kg of air at 1 bar, 15 °C is compressed reversibly adiabatically to a pressure of 4 bars. Calculate the final temperature and the work done on the air. Ans [155, 100.5 $\frac{kJ}{kg}$]

Q4

1 kg of air at 1.02 bar & 20 °C is compressed reversibly according to a law $PV^{1.3} = C$, to a pressure of 5.5 bar. Calculate the work don on the air and the heat flow during the compression.

Ans [133.5 kJ , -33.38 kJ]

Q5

Nitrogen M=28 expands reversible in a perfectly thermally insulated cylinder from 3.5 bars. 200 °c to a volume of 0.09 m³. If the initial volume occupied was 0.03 m³, Calculate the work done during the expansion. Assuming nitrogen to be perfect gas and take $C_v = 0.714$ kJ/kg k Ans [9.31 kJ]

Q6

0.05 kg of Carbon dioxide M=44, occupying a volume of 0.03 m³ at 1.025 bar. is Compressed reversibly until the Pressure is 6.15 bars. calculate the final temperature, the work done on the CO_2, and the heat flow to or from the cylinder walls.

a. when the process is according to a law $PV^{1.4} = C$

b. " " " " " isothermal.

c. " " " " " take place in a perfectly thermally insulated Cylinder. $\gamma = 1.3$.

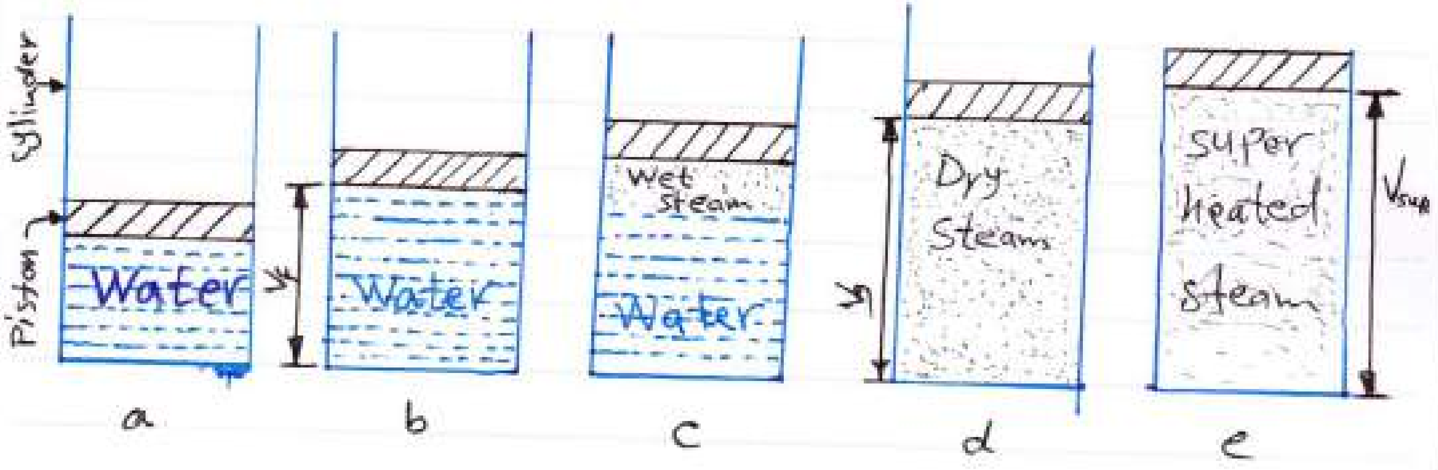
Ans [270 °c , 5.133 kJ , 1.713 kJ , 52.6 °C , 5.5 kJ , -5.5 kJ , 219 °c , 5.25 kJ , 0 kJ].

" Steam and two-phase System "

⇒ Steam :-

Is a vapour of water. and is invisible when pure and dry. It is used as the working substance in the operation of steam engines and steam turbines. Steam does not obey laws of Perfect gases, until it is perfectly dry.

⇒ Formation of Steam at Constant Pressure for water :-



Consider 1 kg of water at 0 °C contained in the Piston-cylinder arrangement as shown in the fig.(a).

The Piston maintain a constant Pressure in the cylinder If we heated the water contained in the cylinder, it will be converted into steam as below :-

1. The volume of water will increase slightly with the increase in temperature as shown in fig (b). It will cause the piston to move slightly upwards and hence work is obtained. This increase in volume of water is generally neglected for all types of calculation.

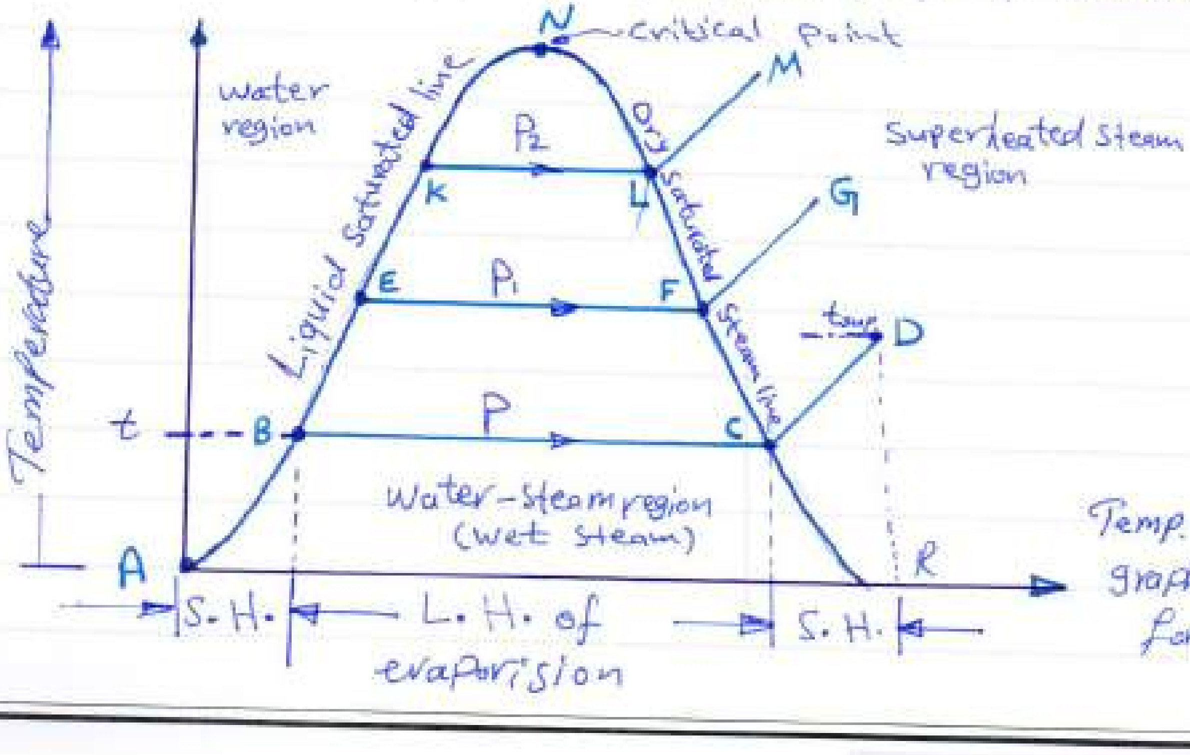
2. On further heating temperature reaches boiling point. The boiling point of water at normal atmospheric pressure of 1.01325 bar is 100 °C, but it increases with the increase in pressure. when the boiling point is reached, the temperature remains constant and the water evaporates, thus pushing the piston up against the constant pressure. Consequently the specific volume of steam increase as shown in fig. (c). At this stage the steam will have some particles of water in suspension and is termed as "wet steam". This process will continue till the whole water is converted into wet steam.

3. On further heating, the particles in suspension will be converted into steam. The entire steam in such a state is termed as "dry steam" or "saturated steam" as shown in fig. (d). Practically the dry steam behaves like a perfect gas.

4. On further heating, the temperature of the steam starting rising. The steam in such a state is termed as Superheated Steam as shown in fig. (e).

Temperature vs. Total Heat Graph during Steam formation:

The Process of steam formation may be represented on a graph, whose abscissa represents the total heat and the vertical cordinate represents the temperature. The point 'A' represents the initial condition of water at 0°C and pressure P (in bar) as shown in fig. below. Line ABCD shows the relation between temperature and heat at a specific pressure of P (in bar).



Temp. – total heat graph during Steam formation.

During the formation of the superheated steam from water at freezing point, the heat is absorbed in the following three stages :-

1. The heating of water up to boiling temp or saturation temperature " t_s " is shown by "AB". The heat absorbed by the water is AP, known as sensible heat (S.H.) or liquid heat or total heat of water.
2. The change of state from liquid to steam is shown by "BC". The heat absorbed during this stage is "PQ" known as latent heat (L.H.) of vaporisation.
3. The superheating process is shown by "CD". The heat absorbed during this stage is "QR" known as Heat of superheat (H.S.). line AR represents the total heat of the superheated steam.

If the pressure is increased "say P_1 bar", the boiling temp also increase. The point "E" represents the boiling temp. or saturation temp at pressure (P_1) and (F) is the point of dry saturated steam. Line "FG" shows the constant pressure process, in which the steam is superheated. The line passing through the points "A, B, E, K" is known as "Saturated Liquid Line" which forms boundary line between water and steam. Similarly a line passing through dry steam points "L, F, C" known as "dry saturated steam line" which forms boundary line between wet and superheated steam.

It may also be noted from the fig., that when the Pressure and Saturation temperature increases, the latent heat of vaporisation decreases. It becomes zero at point " N " when liquid and dry steam lines meet. This Point " N " is known as " Critical Point " and at this Point, the liquid and vapour phases merge. The temperature corresponding to Critical Point " N " is known as Critical ~~Point~~ temperature and the Pressure is known as Critical Pressure. For steam, the Critical temp. is (374.15 °c) and Critical Pressure is (221.2 bars).

Important Terms for Steam :-

1. Wet steam :-

When the steam contains moisture or Particles of water in suspension, it is said to be wet steam. It means that the evaporation of water is not complete and the whole of the latent heat has not been absorbed.

2. Dry Saturated Steam :-

When the wet steam is further heated and it does not contain any suspended particles of water, it is known as dry saturated steam. The dry saturated steam has absorbed its full latent heat and behaves partically, in the same way as a perfect gas.

3. Superheated Steam :-

When the dry steam is further heated at a constant pressure, thus raising its temperature, it is said to be Superheated steam. Since the pressure is constant, therefore the volume of superheated steam increases.

4. Dryness fraction or quality of wet steam :-

It is the ratio of the mass of actual dry steam to the mass of same quantity of wet steam, and generally denoted by «x» Mathematically

$$\boxed{x = \frac{m_g}{m_g + m_f} = \frac{m_g}{m}}$$

m_g = mass of actual dry steam.
m_f = " " water in suspension.
m = " " wet steam.

5. Sensible heat of water :-

It is the amount of heat absorbed by 1 kg of water. when heated at a constant pressure, from the freezing point «0 °C» to the temperature of formation of steam. The sensible heat is also known as Liquid heat.

The specific heat of water at constant pressure is usually take as 4.2 kJ/kg.K .

$Q_{sensible}$ = mass * specific heat * Rise in temp.

$$Q_{sen} = m C_p (T_2 - T_1) = m(h_2 - h_1)$$

$h_2, h_1 \equiv$ Enthalpy of fluid.

6. Latent heat of vaporisation :-

It is the amount of heat absorbed to evaporate 1 kg of water, at it's boiling point or saturation temp. without change of temp. It is denoted by " h_{fg} " and its value depends upon the pressure.

The heat of vaporisation of water or latent heat of steam is 2257 kJ/kg at atmospheric pressure.

It has been experimentally found that the value of h_{fg} decreases as the pressure increases and it is zero at critical pressure.

If the steam is wet with a dryness fraction (x), then the heat absorbed by it during evaporation is $x h_{fg}$.

7. Enthalpy or total heat of steam :-

It is amount of heat absorbed by water from freezing point to saturation temperature plus the heat absorbed during evaporation.

∴ Enthalpy = Sensible heat + Latent heat

It is denoted by h_g and its value for dry saturated steam may be read directly from the steam table

The expression for the enthalpy of wet steam, dry steam and superheated steam are follows:-

i- Wet steam: the enthalpy of wet steam is given by

$$h = h_f + X * h_{fg} \quad ; \quad h_{fg} = h_g - h_f$$

$$h_f = u_f + P V_f$$

ii- Dry Steam: $x = 1$

$$\therefore h = h_f + h_{fg} = h_f + (h_g - h_f) = h_g$$

iii- Superheated Steam: If further add heat to the dry steam, its temperature increases while pressure remaining constant. This increase in temp. shows the superheat stage of the steam. Thus the total heat required for the steam to be superheated.

h_{sup} = Total heat for dry steam + Heat for superheated steam.

$$h_{sup} = h_f + h_{fg} + C_p * (t_{sup.} - t_s) = h_g + C_p (t_{sup.} - t_s)$$

where:

C_p = Specific heat at constant pressure for superheated steam.

$t_{sup.}$ = Temp. of superheated steam

t_s = Saturation temp. at the given pressure.

$(t_{sup} - t_s)$ = degree of superheated

C_p for steam lies between 1.67 to 2.5 kJ/kg.K

8. Specific volume of steam: It is the volume occupied by the steam per unit mass at given temperature and pressure.

The expression for the volumes occupied by steam are as follows:

1- Wet steam: Consider 1 kg of wet steam of dryness fraction X. Known this steam will has X kg of dry steam and $(1-X)$ kg of water. let V_f be the volume of 1 kg of water, then

Volume of 1 kg of wet steam $= X V_g + (1-X) V_f$

Since V_f is very small as compared to V_g, therefore the $(1-x) V_f$ may be neglected

$\therefore V = x * V_g \quad m^3/kg$

2- Dry steam: We known at dry saturated the mass of water in suspension is zero and dryness fraction is unity.

$\therefore V = V_g$

3. Superheated steam

$$\frac{V_{sup.}}{T_{sup.}} = \frac{V_g}{T} \Longrightarrow V_{sup} = \frac{V_g * T_{sup}}{T}$$

Ex. : Find the enthalpy , internal energy and the specific volume of saturated water at 2 bars.

Sol.

$P = 2$ bars

from steam table at 2 bars.

$h = h_f = 505\ kJ/kg$; $u = u_f = 505\ kJ/kg$; $v_f = 0.001\ \frac{m^3}{kg}$

Ex. : Find the specific volume, enthalpy , entropy and internal energy of vapor at 18 bars and dryness fraction or quality is (0.9).

Sol.

$X = 0.9 \Rightarrow$ Wet steam

from steam table at 18 bars

$V_g = 0.114\ m^3/kg$; $h_f = 885\ \frac{kJ}{kg}$; $h_{fg} = 1912\ \frac{kJ}{kg}$; $U_f = 883\ \frac{kJ}{kg}$

$U_g = 2598\ kJ/kg$; $S_f = 2.398\ kJ/kg.K$; $S_{fg} = 3.981\ kJ/kg.K$

$V = X\,V_g = 0.9 * 0.114 = 0.0994\ m^3/kg$

$h = h_f + X\,h_{fg} = 885 + 0.9 * 1912 = 2605\ kJ/kg$

$u = u_f + X\,u_{fg} = 883 + 0.9 * (2598 - 883) =$

$S = S_f + X\,S_{fg} = 2.398 + 0.9 * 3.981 = 5.98\ kJ/kg.K$

Ex. : Steam is initially saturated at 6 bars. Find the specific volume, internal energy, enthalpy and entropy.

Sol.

Saturated steam $\Rightarrow x = 1$

from table at $P = 6$ bars.

$v = v_g = 0.3156\ m^3/kg$, $h = h_g = 2757\ kJ/kg$; $u = u_g = 2563\ kJ/kg$

$S = S_g = 6.761\ kJ/kg$

Ex. : Steam at 5.3 bars and 200 °c. Find the specific volume and internal energy.

Sol.

Steam at $P = 5.3$ bars. , $T = 200°c$

from table at 5.3 bars $\Rightarrow t_s =$

$\because T = 200°c > t_s =$

$\therefore$ the state is superheated steam.

By interpolation at $P = 5.3$ bars and $T = 200°c$

$$\frac{v - 0.4252}{0.3522 - 0.4252} = \frac{5.3 - 5}{6 - 5} \Rightarrow v = 0.4033\ \frac{m^3}{kg}$$

$$\frac{u - 2644}{2640 - 2644} = \frac{5.3 - 5}{6 - 5} \Rightarrow u = 2642\ kJ/kg$$

Ex.: Determine the condition of steam in the following cases:

1 - A a pressure of 10 bars and temperature 200 °C

2 - " " " " " " Volume 0.175 m³/kg.

Sol.

1- $P = 10$ bars , $T = 200$ °C

from st. table at $P = 10$ bars $\Rightarrow t_s = 179.9$ °C

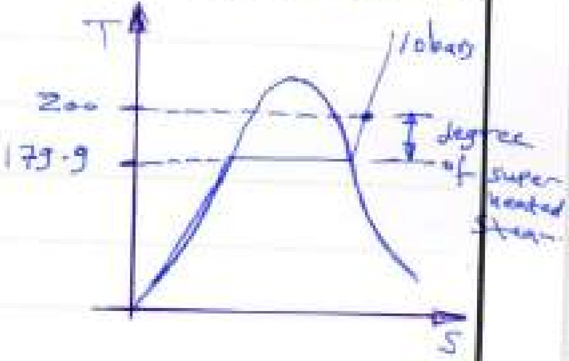
$\therefore T > t_s \Rightarrow \therefore$ the state is superheated steam.

at $P = 10$ bars

$v_g = 0.194$ m³/kg , $h_f = 762.6$ KJ/Kg

the degree of Superheated steam $= T - t_s$

$= 200 - 179.9 = 20.1$ °C



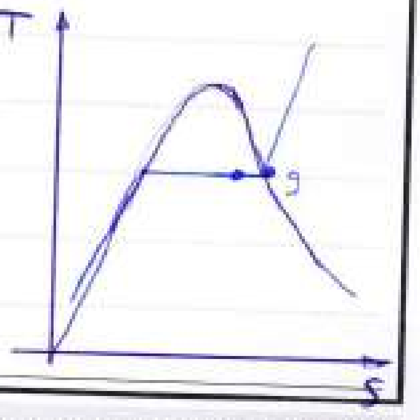
2- at 10 bars $v_g = 0.194$ m³/kg

$V = 0.175$ m³/kg is given

$\therefore V_g > V \Rightarrow$ the state is wet steam.

$V = x \cdot V_g \Rightarrow x = \frac{V}{V_g}$

$\therefore x = \frac{0.175}{0.194} = 0.902$



Ex.: Find the dryness fraction, the specific volume and internal energy of steam at 7 bar and enthalpy 2600 kJ/kg

Sol.

at 7 bars from steam table

$h_g = 2764$ kJ/kg , $h_f = 697$ kJ/kg

$\because h_g > h > h_f \Rightarrow$ The state is wet steam.

$h = h_f + x * h_{fg} = 2600 = 697 + x * (2764 - 697)$

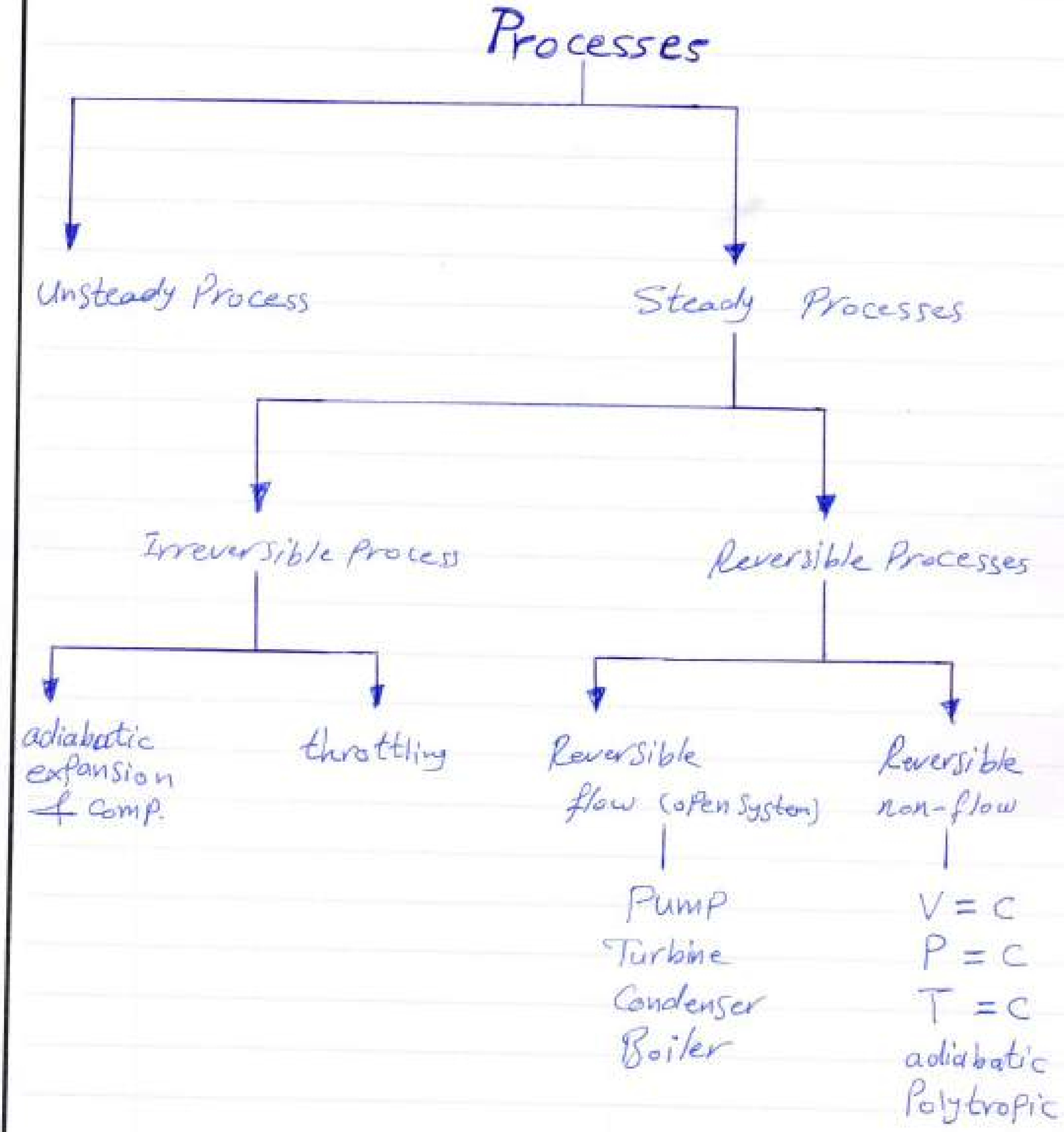
$x = 0.921$

$v = x * v_g \Rightarrow$ at 7 bars from s. t. $v_g = 0.2728$ m³/kg

$v = 0.921 * 0.2728 = 0.2515$ m³/kg

$u = u_f + x(u_g - u_f) = 696 + 0.921(2573 - 696)$

$= 2420$ kJ/kg


Processes
Unsteady Process
Steady Processes
Irreversible Process
Reversible Processes
adiabatic expansion & comp.
throttling
Reversible flow (open System)
Reversible non-flow
Pump
Turbine
Condenser
Boiler
V = C
P = C
T = C
adiabatic
Polytropic

Reversible and Irreversible Processes :-

1. Closed System (Reversible)

A: Constant Volume Process (Isometric Process)

from non-flow energy equation

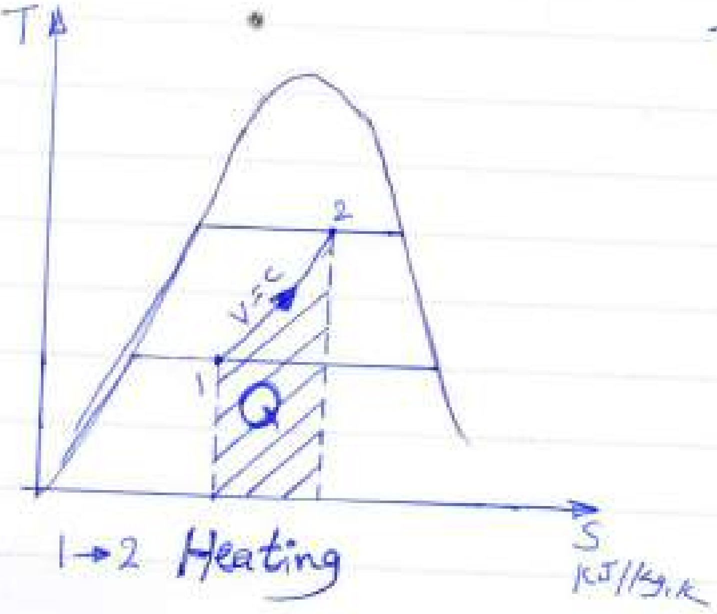
$$Q = W + \Delta U$$

$$V = c \Rightarrow W = 0$$

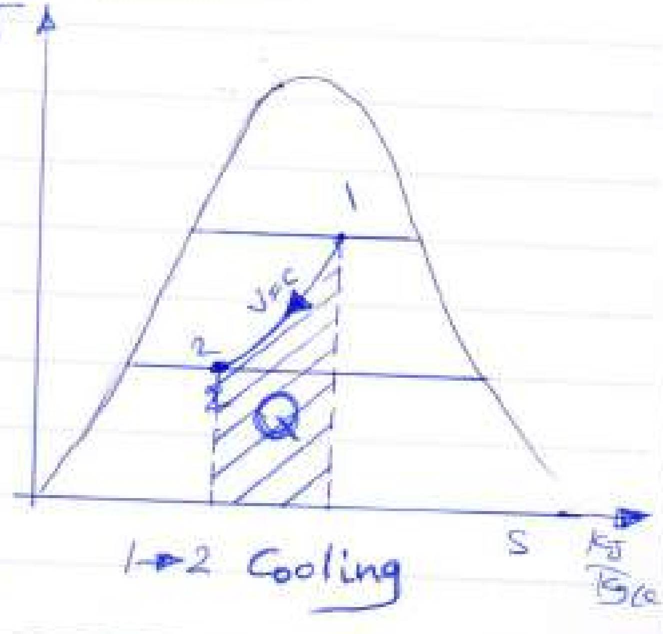
$$\therefore Q = \Delta U = U_2 - U_1$$

$$h = u + Pv$$

$$\therefore \boxed{Q = u_2 - u_1 = (h_2 - h_1) - v(P_2 - P_1)}$$



1→2 Heating



1→2 Cooling

Ex.: A vessel having a volume of 5 m^3 contains 0.05 m^3 of saturated liquid water and 4.95 m^3 of saturated water vapour at 0.1 MPa. Heat is transfered until the vessel is filled with saturated vapour. Determine the heat transfer for this process.

Sol.:

Condition 1 wet steam.

at 0.1 MPa = 100 kpa

$v_f = 1.0432 \times 10^{-3}$ m^3/kg , $v_g = 1.694$ m^3/kg

$V = m * v \Rightarrow m = V/v$

$\therefore m_L = \frac{V_L}{v_L} = \frac{0.05}{1.043 \times 10^{-3}} = 47.93$ kg

$m_g = \frac{V_g}{v_g} = \frac{4.95}{1.694} = 2.92$ kg

$\therefore X = \frac{m_g}{m_t} \Rightarrow \therefore X = \frac{2.92}{50.85} = 0.0574$

$\therefore U_1 = U_{f_1} + X U_{fg_1}$ at 100 kpa

$U_1 = 417.36 + 0.0574 * 2088.7 = 521.8$ kJ/kg

Condition 2 V = constant , Dry saturated steam.

$v_1 = \frac{V}{m} = \frac{5}{50.85} = 0.09833$ m^3/kg $= v_2$ at dry sat. st.

from steam table and at $v_2 = 0.09833$ and dry saturated we will found $U_2 = U_{g_2}$

By Interpolation

v_g	u_g
0.09963	2600.25
0.09833	u_2
0.08875	2601.98

$$\frac{u_2 - 2600.25}{2601.98 - 2600.25} = \frac{0.09833 - 0.09963}{0.08875 - 0.09963}$$

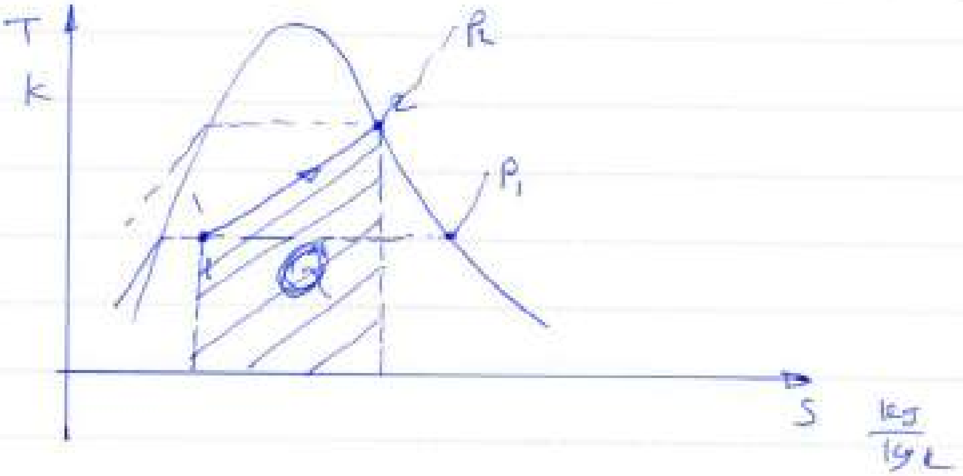
$$\Rightarrow u_2 = 2600.465 \text{ kJ/kg}$$

& $Q = W + \Delta U$, $W = 0$ $V = c$

$Q = U_2 - U_1 \Rightarrow Q = 2600.465 - 521.8$

$Q = 2078.665$ kJ/kg

$Q = 50.85 * 2078.665 = 105700.1$ kJ



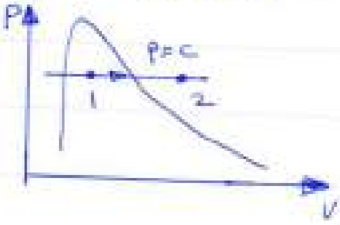
B. Constant Pressure Process :-

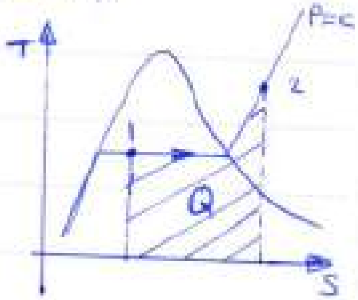
from NFEE

$Q = W + \Delta U \quad , W = \int P\,dv = P(V_2 - V_1)$

$Q = (P_2V_2 - P_1V_1) + (U_2 - U_1) = H_2 - H_1$

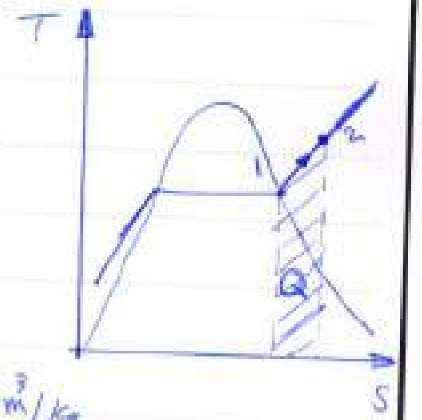
$$\therefore \boxed{Q = m(h_2 - h_1)}$$





Ex. :- 0.05 kg of a dry saturated steam is heated at a constant pressure of 2 bars until the volume occupied is 0.0658 m³. Calculate the heat supplied and the work done.

Sol.



$m = 0.05$ kg , $X = 1$ $P = 2$ bars $= C$

Condition 1 : dry sat. $X = 1$, $P = 200$ kPa
from steam table at dry-sat.
and at $P = 200$ kPa
$h_1 = h_g = 2707$ kJ/kg , $v_1 = v_{g_1} = 0.8856$ m³/kg

Condition 2 : $P_2 = P_1 = \text{Const.} \Rightarrow P_2 = 200$ kPa ; $V_2 = 0.0658$ m³
$V_2 = m v_2 \Rightarrow v_2 = \frac{0.0658}{0.05} \Rightarrow v_2 = 1.316$ m³/kg
∴ from st. table at $P = 200$ kPa and $v_{g_2} = 0.8856$ m³/kg
∴ $v_2 > v_{g_2} \Rightarrow$ the state is superheated st.

from St. table superheated st. at 200 kPa and $v_2 = 1.316 \ \frac{m^3}{kg}$

$h_2 = 3072$ kJ/kg

$Q = m(h_2 - h_1) = 0.05(3072 - 2707) = 18.25$ kJ

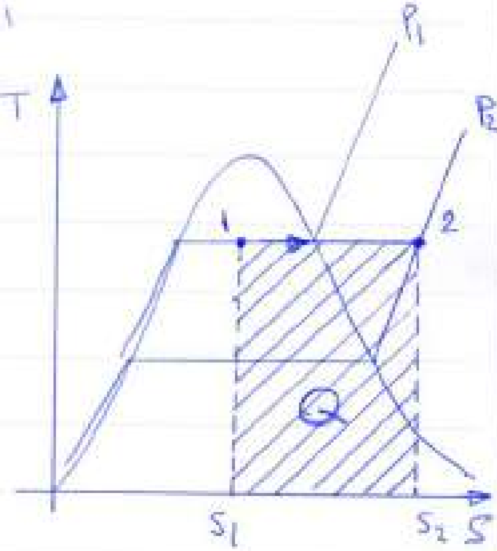
$W = mP(v_2 - v_1) = 0.05 \times 200(1.316 - 0.885) = 4.304$ kJ

C. Constant temperature (Isothermal Process)

from NFEE

$Q = W + \Delta U \quad ; \Delta U = U_2 - U_1$

$Q = mT(S_2 - S_1)$ kJ



Ex.:

Steam at 7 bars and quality 0.9 , expands in cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bars. Calculate the change in enthalpy of steam and the heat supplied during process and the work done, sketch (T-S).

Sol.:

Condition 1: $P_1 = 7$ bars $= 700$ kPa $\quad x_1 = 0.9 \Rightarrow$ wet st.

from St. table at 700 kPa $T = t_s = 165$ °C

$h_{f_1} = 697$ kJ/kg ; $h_{fg_1} = 2067$ kJ/kg ; $h_1 = h_{f_1} + x_1 h_{fg_1}$

$h_1 = 697 + 0.9(2067) = 2557.3$ kJ/kg

$u_{f_1} = 696$ kJ/kg ; $u_{g_1} = 2573$ kJ/kg $u_1 = 696 + 0.9(2573 - 696)$

$\therefore u_1 = 2385.3$ kJ/kg

$s_{f_1} = 1.992$ kJ/kg ; $s_{fg_1} = 4.717$ kJ/kg.K

$s_1 = 1.992 + 0.9 * 4.177 = 6.237$ kJ/kg.K

<u>Condition 2 :</u> $P_2 = 1.5$ bar $= 150$ kPa

$T_2 = T_1 = 165$ °C

from st. table at 150 kPa $t_{s2} = 111.4$ °C

$T_2 > t_{s2} \Rightarrow$ The state (2) superheated steam.

By Interpolation:-

at 100 kPa

T °C	h	u	s
150	2776.38	2582.75	7.6133
165	h_2	u_2	s_2
200	2875.27	2658.05	7.8342

$$\frac{165 - 150}{200 - 150} = \frac{h_2 - 2776.38}{2875.27 - 2776.38} = \frac{u_2 - 2582.75}{2658.05 - 2582.75} = \frac{s_2 - 7.6133}{7.8342 - 7.6133}$$

$\therefore h_2 = 2806$ kJ/kg , $u_2 = 2605.34 \frac{kJ}{kg}$; $s_2 = 7.67957 \frac{kJ}{kg.K}$

at 200 kPa

T	h	u	s
150	2768.8	2576.87	7.2795
165	h_2	u_2	s_2
200	2870.46	2654.39	7.5066

$\Rightarrow h_2 = 2799.3$ kJ/kg

$u_2 = 2600.12$ kJ/kg

$s_2 = 7.34763$ kJ/kg

NOW: h_2 at 150 kPa & 165°C

$$h_2 = \frac{h_{2,100kPa} + h_{2,200kPa}}{2} = \frac{2806 + 2799.3}{2} = 2802.65 \frac{kJ}{kg}$$

$$U_2 = \frac{U_{2,100kPa} + U_{2,200kPa}}{2} = \frac{2605.34 + 2600.12}{2} = 2602.73 \frac{kJ}{kg}$$

$$S_2 = \frac{S_{2\,100kPa} + S_{2\,200kPa}}{2} = \frac{7.6795 + 7.34763}{2} = 7.5133 \frac{kJ}{kgK}$$

$\therefore \Delta h = 2802.65 - 2557.3 = 245.35$ kJ/kg

$Q = T * \Delta S = T * (S_2 - S_1)$

$= (165 + 273)(7.5133 - 6.237) = 559 \frac{kJ}{kg}$

$\Delta u = u_2 - u_1 = 2602.73 - 2385.3$

$\Delta u = 217.43$ kJ/kg

From NFEE $Q = W + \Delta u \Rightarrow W = Q - \Delta u$

$\therefore W = 559 - 217.43 = 341.57$ kJ/kg • Ans

D. The hyperbolic process $PV = C$

$P_1V_1 = P_2V_2 \Rightarrow$ work done $= W = PV \ln \frac{v_2}{v_1}$

from NFEE

$Q = W + \Delta u = (u_2 - u_1) + Pv\, l \frac{v_2}{v_1}$

$= (h_2 - P_2v_2) - (h_1 - P_1v_1) + Pv\, l \frac{v_2}{v_1} \quad ; P_1V_1 = P_2V_2$

$= h_2 - h_1 + Pv \ln \frac{v_2}{v_1}$

$$\therefore \boxed{Q = (h_2 - h_1) + Pv \ln \frac{v_2}{v_1}}$$

Ex.:- A quantity of dry Saturated Steam occupies 0.2634 m^3 at 1.5 MPa. Determine the final condition of the steam if it is compressed until the volume is halved:

a) if the compression is carried out in an isothermal Pr.

b) " " " " follows the law $PV=c$.

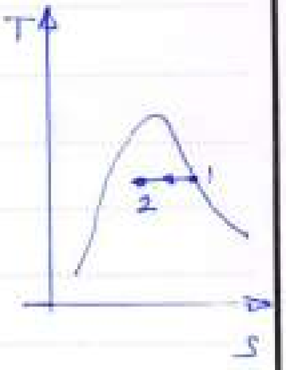
Sol.:-

a) from St. table at 1.5 Mpa $\Rightarrow v_g = 0.1317 \frac{m^3}{kg}$

$$m = \frac{V}{v} = \frac{0.2634}{0.1317} = 2 \text{ kg}$$

$$v_2 = \frac{v_1}{2} = \frac{v_{g_1}}{2} = \frac{0.1317}{2} = 0.0659 \, m^3/kg$$

$$v_2 = x_2 * v_{g_2} \Rightarrow x_2 = \frac{v_2}{v_g} = \frac{0.0659}{0.1317} = 0.5$$



$$h_2 = h_{f_2} + x_2 h_{fg_2}$$

from steam table at $P = 1.5$ MPa

$h_f = 844.7$ kJ/kg ; $h_{fg} = 1945.2$ kJ/kg

$\therefore h_2 = 844.7 + 0.5(1945.2) = 1817.3$ kJ/kg

$H_2 = m * h_2 = 2 * 1817.3 = 3634.6$ kJ

Heat loss $= 0.5 * h_{fg} = 0.5 * 1945.2 = 972.6$ kJ/kg.

" $= 2 * 972.6 = 1945.2$ kJ

b) $PV = c$

then $P_1 V_1 = P_2 V_2 \Rightarrow P_2 = P_1 * \frac{V_1}{V_2} = 1.5 * 2 = 3$ MPa

Specific volume after compression $v_2 = \frac{v_1}{2} = \frac{v_g}{2} = 0.0659 \frac{m^3}{kg}$

from St. table

at $P_2 = 3$ MPa, $v_g = 0.0666$ m³/kg, $h_f = 1008.4$ kJ/kg

$h_{fg} = 1793.9$ kJ/kg.

$v_g > v_2 \Rightarrow$ wet steam.

$v = x v_g \Rightarrow x = \frac{v_2}{v_g} = \frac{0.0659}{0.0666} = 0.989$

$\therefore h_2 = h_f + x h_{fg} = 1008.4 + 0.989 * 1793.9 = 2782.6 \frac{kJ}{kg}$

$H_2 = m h_2 = 2 * 2782.6 = 5565.2$ kJ

E. The Polytropic Process $PV^n = c$

Here the steam is assumed to be expanded or compressed according to the Law $PV^n = c$

from non-flow energy eq.

$Q = W + \Delta U$; $W = \frac{P_1 V_1 - P_2 V_2}{n-1}$

$\Delta U = U_2 - U_1 = (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$

$= (h_2 - h_1) - (P_2 V_2 - P_1 V_1)$

$$q = \frac{P_1v_1 - P_2v_2}{n-1} + (h_2 - h_1) - (P_2v_2 - P_1v_1)$$

$$= (h_2 - h_1) + \frac{P_1v_1 - P_2v_2}{n-1} + (P_2v_2 - P_1v_1)$$

$$= (h_2 - h_1) + \frac{P_1v_1 - P_2v_2 + (n-1)(P_2v_2 - P_1v_1)}{n-1}$$

$$= (h_2 - h_1) + \frac{(P_1v_1 - P_2v_2)(1 + n - 1)}{n-1}$$

$$= (h_2 - h_1) + \frac{n \cdot (P_1v_1 - P_2v_2)}{n-1} = (h_2 - h_1) + n \cdot w$$

$$\therefore \boxed{Q = (H_2 - H_1) + n \cdot W}$$

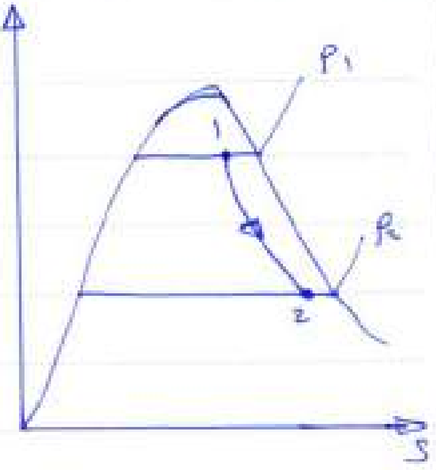
Ex.: A quantity of steam at a pressure of 2.1 MPa and 0.9 dry occupies a volume of 0.2562 m^3. It is expanded according to the Law $PV^{1.25} = C$, to a pressure of 0.7 MPa. Determine: a- the mass of steam present. b- the external work do c- the change of internal energy. d- the heat exchange between the steam and surrounding.

Sol.:-

a- $m = \frac{V}{v}$; $V = 0.2562 \ m^3$

from steam table at 2100 kPa $\Rightarrow v_f = 1.181 \times 10^{-3} \ \frac{m^3}{kg}$, $v_g = 0.0949 \ \frac{m^3}{kg}$

$\therefore v_1 = 1.181 \times 10^{-3} + 0.9(0.0949 - 1.181 \times 10^{-3}) = 0.0856 \ m^3/kg$



$\therefore m = \dfrac{V}{v} = \dfrac{0.2562}{0.0856} = 3$ kg

b. $w = ?$

$$w = \frac{P_1 v_1 - P_2 v_2}{n-1}$$

$$P_1 v_1^{1.25} = P_2 v_2^{1.25} \Rightarrow v_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.25}} * v_1$$

$$v_2 = \left[\frac{2.1}{0.7}\right]^{1/1.25} * 0.0856 \Rightarrow v_2 = 0.206 \text{ m}^3/\text{kg}.$$

$$\therefore w = \frac{2100 * 0.0856 - 700 * 0.206}{1.25 - 1} = 143.36 \text{ kJ/kg}$$

$$W = m * w = 3 * 143.36 = 430 \text{ kJ}.$$

c. $\Delta U = U_2 - U_1 = ?$

at $p_1 = 2100$ kpa

$u_{f_1} = 917.5$ kJ/kg ; $u_g = 2598.2$ kJ/kg ; $x_1 = 0.9$

$u_1 = 917.5 + 0.9(2598.2 - 917.5) = 2430$ kJ/kg

at $P_2 = 700$ kpa

$v_2 = 0.206$ m³/kg ; $v_g = 0.273$ m³/kg

$\therefore v_g > v_2 \Rightarrow$ The state is wet steam.

$v_2 = v_{f_2} + x_2 v_{fg_2} \Rightarrow 0.206 = 1.108 \times 10^{-3} + x_2 (0.273 - 1.108 \times 10^{-3})$

$x_2 = 0.753$; $u_{f_2} = 696.3$ kJ/kg ; $u_{g_2} = 2571.1$ kJ/kg

$u_2 = 696.3 + 0.753(2571.1 - 696.3) = 2108$ kJ/kg

$\therefore \Delta U = m(u_2 - u_1) = 3 * (2108 - 2430) = -966$ kJ.

d. $Q = W + \Delta U = 430 - 966 = -536$ kJ. (loss to the surrounding)

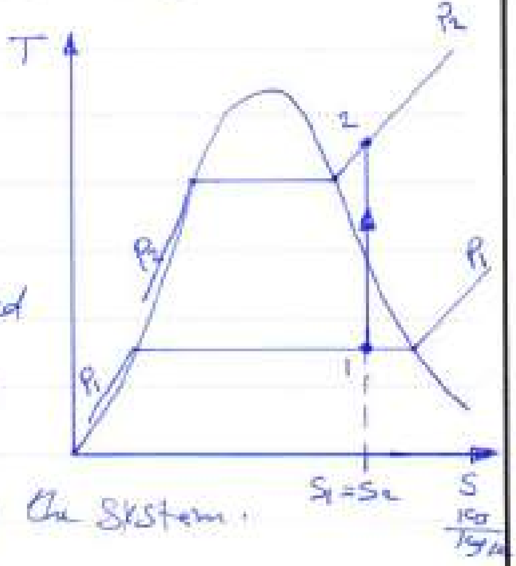
F. Adiabatic Process: "$Q = 0$"

An adiabatic process is one in which no heat is transferred to or from the fluid during the process.

from NFEE

$$Q = W + \Delta U \quad , \quad Q = 0 \Rightarrow W = -\Delta U$$

$$W = U_1 - U_2$$



Ex.:-

1 kg of steam at 10^2 bars ~~kPa~~ and 375 °C expands reversibly in a perfectly insulated cylinder behind a piston until the pressure is 38 bars and the steam is then dry saturated. Calculate the work done by the system.

Sol.

$Q = W + \Delta U$, $Q = 0$ (Perfectly insulated)

$\therefore W = U_1 - U_2$

at 10^4 kPa $t_s = 311$ °C , $T > t_s \Rightarrow$ The state is superheated

$\therefore$ from steam table at 100 kPa & 375 °C

$U_1 = 2621.78$ kJ|kg

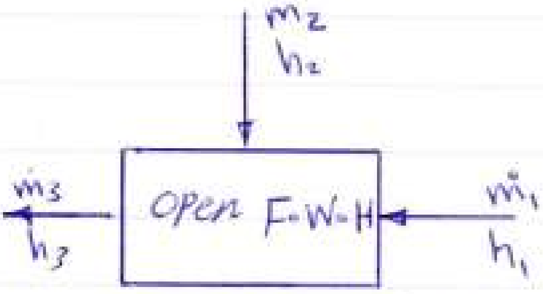
at $P_2 = 3800$ kPa , dry $\Rightarrow U_2 = U_{g_2} = 2608$ kJ|kg

$W = \dot{m} * w$

$= 1 * (U_1 - U_2) = 1 * (2621 - 2608) = 13$ kJ

2. Irreversible Processes :-

1. Adiabatic mixing :



open feedwater heater

from flow eq.

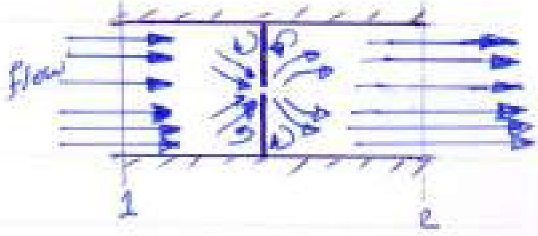
$H_1 + H_2 = H_3$

$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$

$\dot{m}_1 C_{p_1} T_1 + \dot{m}_2 C_{p_2} T_2 = (\dot{m}_1 + \dot{m}_2) C_{p_3} h_3$

2. Throtting :



$h_1 + \frac{1}{2} C_1^2 + q = h_2 + \frac{1}{2} C_2^2 + W$

$Q = 0 \quad , \quad W = 0 \quad ; (C_1^2 - C_2^2)/2 \simeq 0$

$\therefore \ h_1 = h_2 \Rightarrow C_{p_1} T_1 = C_{p_2} T_2$

Ex. :

Steam at 19 bars is throttled to 1 bars and the temperature after throttling is found to be 150 °c, Calculate the initial dryness fraction of the steam.

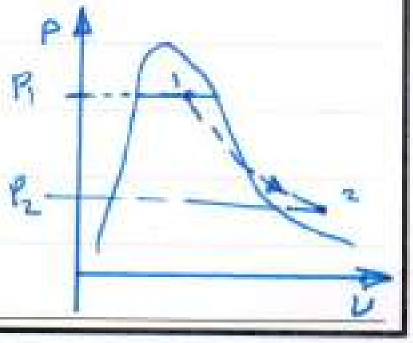
Sol.

$\Delta h = 0 \quad h_1 = h_2$

from st. table at 1 bar & 150 °c $h_2 = 2776.$ $\frac{8}{\frac{60}{19}}$

$h_2 = h_1 + X_1 h_{fg_1} \Rightarrow 2776.36 = 897 + X * (1910$

$\Rightarrow X_1 = 0.989$



Ex. 8 Steam at 1.5 MPa and 250 °c flowing at 1.5 kg/s is throttled to 700 kPa and the mixed with a steam at 700 kPa also 0.97 dryness fraction with a flow rate of 3.6 kg/s find the condition of resulting steam.

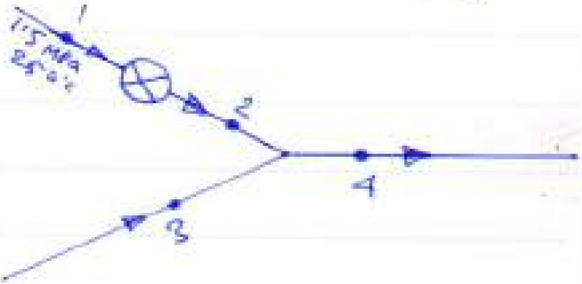
Sol.:

at 1.5 MPa = 1500 kPa, from St. table

$t_{s_1} = 198.32$ °c $\Rightarrow$ ∴ $T_1 > t_{s_1}$; $250 > 198.32 \Rightarrow$ The state is ~~[illegible]~~ is superheated steam.

from steam table of superheated steam $h_1 = 2925$ kJ/kg

$h_1 = h_2$ throttled.



$\dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$

Point ③ $x_3 = 0.97$, $P_3 = 700$ kPa

from st. table at 700 kPa

$h_f = 697$ $\frac{kJ}{kg}$, $h_{fg} = 2067$ kJ/kg

$h_3 = 697 + 0.97(2067) = 2702$ kJ/kg

$$h_4 = \frac{\dot{m}_2 h_2 + \dot{m}_3 h_3}{\dot{m}_2 + \dot{m}_3} = \frac{1.5 * 2925 + 3.6 * 2702}{1.5 + 3.6} = 2767.6 \text{ kJ/kg}$$

at 700 kPa $h_g = 2763.5$ kJ/kg , $t_{s_2} = 164.97$ °c

$h_4 > h_g \Rightarrow$ Superheated steam.

from steam table $T_4 = 166.64$

∴ Degree of Superheated = 166.64 − 164.92 = 1.67 °c

Ex. : The mass flow rate of steam turbine is 1.5 kg/s and the heat transfer from the turbine is 8.5 kW. The following data are known for steam entering and leaving turbine. Find the power output of turbine.

	P(MPa)	t(°c)	X	(m/s)	Z(m)
input	2	350	–	50	6
output	0.1	–	1	200	3

Sol.

from flow energy equation.

$gz_1 + h_1 + \frac{1}{2}c_1^2 + q = gz_2 + h_2 + \frac{1}{2}c_2^2 + w$

$w = g(z_1 - z_2) + (h_1 - h_2) + \frac{1}{2}(c_1^2 - c_2^2) + \frac{q}{\dot{m}}$

Point ①

$P_1 = 2000$ kPa $T_1 = 350$ °c , $t_{s1} = 212.42$ °c

$T_1 > t_{s1} \Rightarrow$ superheated st. $\Rightarrow h_1 = 3136.96$ kJ/kg

Point ② $P_2 = 100$ kPa $x = 1$ $h_2 = h_{g2} = 2675.5$ kJ/kg

$\therefore W = 9.81(6-3) \times 10^{-3} + (313696 - 2675.5) + \frac{1}{2}(50^2 - 200^2) \times 10^{-3} - \frac{8.5}{1.5}$

$w = 437.11$ kJ/kg

Power $= W \times \dot{m} = 1.5 \times 437.11 = 656$ kW.

" Sheet No. 4 "

Q1 : Determine the amount of heat which Should be supplied to 2 kg of water at 25 °c to Convert it into steam at 5 bars and 0.9 dry. [4864.52 kJ]

Q2 : A water Storage tank Contains Liquid and vapour in equilibrium at 110 °C. The distance from bottom of the tank to the Liquid level is 8 m. What is the absolute Pressure at the bottom of the tank. [218 kPa]

Q3 : Find the specific volume and specific enthalpy of 1 kg of Steam at 20 bars when
a- it is superheated, its temperature being 400 °c.
b- " " wet, its dryness fraction being 0.9.

Q4 : Saturated water vapour at 60 °c has it's Pressure decrease to increase the volume by 10% keeping the temp. Constant. To what Pressure Should be expands.
[18.9 kPa]

Q5 : 1 kg of water at 47.8 °c is heated under Constant Pressure of 13.7 bars Until it is Converted into Steam with 300 °c. Calculate the amount of heat supplied during Superheating and the total heat.

Q6: 5 kg of Steam at pressure of 4.9 bar is produced from water at 20°c. Determine the amount of heat Supplied if the Steam is 0.9 dry. [12263 kJ]

Q7: A closed Vessel of 0.6 m^3 capacity contains dry saturated Steam at 3.6 bars. The vessel is Cooled Until the Pressure is reduced to 2 bars. Calculate:

a – the mass of Steam in the Vessel.

b – the final dryness of the Steam.

c – the amount of heat transferred during the Cooling Pro.

[1.18 kg , 0.576 , −918.6 kJ].

Q8: ~~1 kg of water~~

A tank of 2 m^3 Volume contains Saturated ammonia at a temp. of 40°c, the tank Contains 50% Liquid and 50% Vapour by Volume. Find the specific Volume, enthalpy and internal energy of the mixture.

[3.369×10^{-3} m^3/kg , 232908 kJ , 229816 kJ]

The Second Law of Thermodynamics

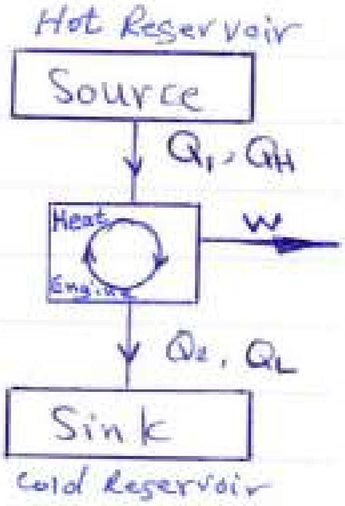
According to the first Law of thermodynamics, when a System undergoes a complet cycle then the net heat supplied is equal to the net work done.

$$\oint dQ = \oint dw$$

The Second law of thermodynamics indicates that although the net heat supplied in a cycle is equal to the net work done, the gross heat supplied must be greater than the net work done, Some heat must always be rejected by the system.

The Heat Engine :-

A heat engine is a system operating in a complete cycle and developing net work from a supply of heat. The Second Law implies that a Source of heat supply and a sink for the rejection of heat are both necessary. Since some heat must always be rejected by the system.



The heat supplied from the Source is Q_1, the work done is W, and the heat rejected is Q_2.

The Second Law of Thermodynamics

القانون الثانى للديناميك الحرارى:

مقدمة: ذكرنا سابقا نص القانون الاول للديناميك الحراري وهو عند مرور نظام ما بدورة كاملة فان صافى الحرارة المضافة من المحيط الى النظام يساوى صافى الشغل المنجز من النظام على المحيط. اى ان

$$\sum dQ = \sum dw$$

، ، النص العام للقانون الثانى لديناميك الحراري هو:

على الرغم من ان صافي الحرارة المضافة الى النظام المار بدورة كاملة يساوي صافي الشغل المنجز، الا ان اجمالي الحرارة المضافة الى النظام يجب ان تكون اكبر من صافى الشغل المنجز، اي يجب ان يكون هنالك مصدر لأضافة الحرارة واخر تسحب الحرارة.

لذلك لاعطاء مفهوم شامل للقانون الثاني لديناميك الحرارة يجب مناقشة كل من الماكنة الحرارية و المضخة الحرارية:

الماكنة الحرارية: هي نظام يعمل بدورة كاملة ويقوم بانتاج صافي الشغل عن طريق الحرارة المضافة الى النظام.

حسب النص العام للقانون الثاني : فأنه يجب ان يكون هنالك مصدر لتجهيز الحرارة واخر لسحبها كما في الشكل -1-

يمثل الشكل -1- ماكنة حرارية تضاف الحرارة من المصدر بمقدار Q_1 ، ينجز الشغل من قبل الماكنة الحرارية بمقدار W وتطرح الحرارة الى المصب بمقدار Q_2.

من القانون الاول للديناميك الحراري:

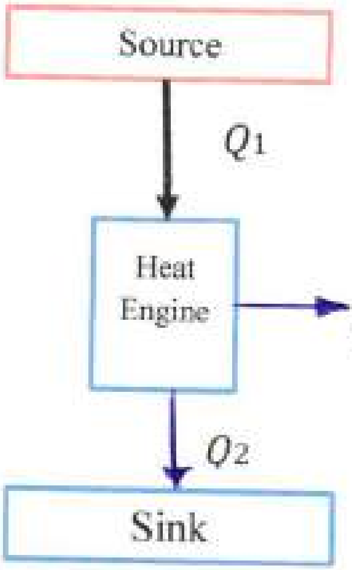
$$\sum dQ = \sum dw$$

اي ان

$$\sum dQ = Q_1 - Q_2$$

$$Q_1 - Q_2 = W$$

اذن يجب ان تكون $Q_1 > W$ ومن هنا نستنتج انه يجب ان تكون هناك كفاءة حرارية للماكنة الحرارية



شكل -1-

By the first law of thermodynamics, in a complete cycle
Net heat supplied = net work done

$$\Sigma dQ = \Sigma dw$$

$$Q_1 - Q_2 = W$$

By the second law, the gross heat supplied must be greater than the net work done

$$Q_1 > W$$

- The thermal efficiency :-

The thermal efficiency of a heat engine is defined as the ratio of the net work done in the cycle to the gross heat supplied in the cycle.

thermal efficiency, $\eta = \dfrac{W}{Q_1}$

$W = Q_1 - Q_2$

$$\therefore \quad \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad . \eta < 100\%$$

From the definition of heat, a temperature difference is necessary for heat to flow, therefore that the source of heat must be at a higher temperature than the Sink.

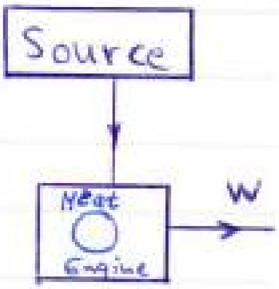
The Source can be thought of as a heat reservoir and the Sink as a cold reservoir.

There are two Statements of the Second Law:
1. The Kelvin - Planck Statement.
2. The clausius Statement.

1. The Kelvin - Planck Statement:

It is impossible for any system to operate in a thermodynamics cycle and deliver a net amount of work to its surrounding while exchanging heat with a single heat reservoir.

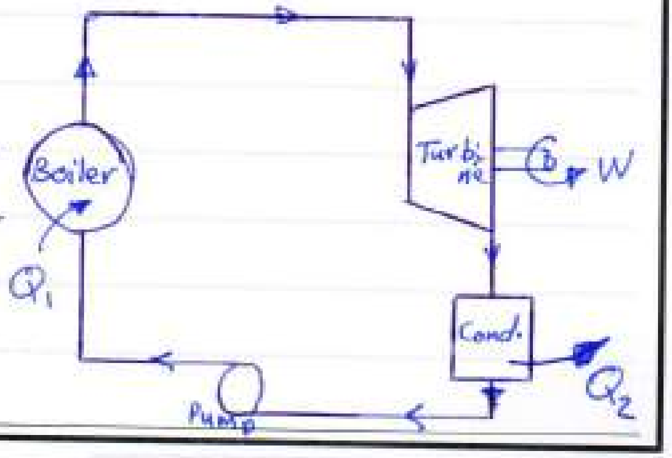
The Kelvin - Plank Statement of the Second law:
It is not possible to have the following System with $Q_2 = 0$



we cannot convert all of the heat from a heat reservoir at T_H to work

The fig. represente as simple steam cycle.

Q_1: heat is supplied in the boiler.
Q_2: heat is rejected in a condensor.
W : Work is developed in the steam turbine or engine

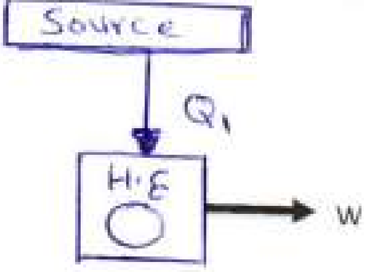


يجب ان يكون هنالك فرق في درجات الحرارة كي تنتقل الحرارة من جسم الى اخر، لذلك يجب ان يكون المصدر عند درجة حرارة اعلى من المصب كي يحدث انتقال حرارة.

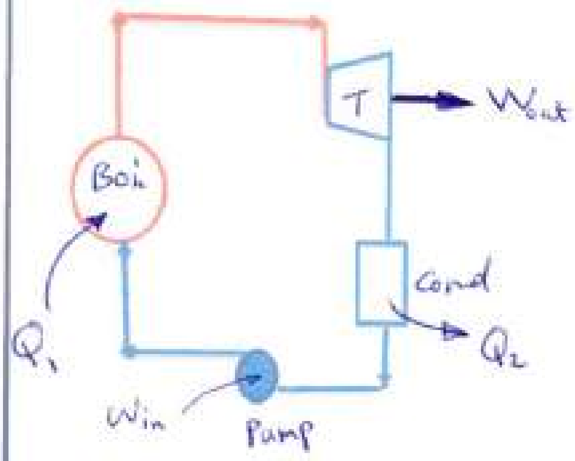
من هنا يمكن كتابة صيغة خاصة للقانون الثاني لديناميك الحرارة وخاصة بالماكنة الحرارية وتسمى صيغة كلفن بلانك

صيغة كلفن – بلانك kelvin – Planck statement

يستحيل انشاء وسيلة ما تعمل ضمن دورة ولا يكون لها تاثير سوى سحب الحرارة من خزان ساخن وحيد عند درجة حرارة ثابتة وانجاز شغلا مساو لهذة الحرارة. اي يجب ان يكون هنالك مصب للحرارة كي تعمل الدورة.



الشكل لايجوز حسب صيغة كلفن بلانك.



من الامثلة الجيدة للماكنة الحرارية هو دورة البخار البسيطة. حيث يمثل الوقود المحروق في المرجل الحرارة المضافة Q_1 والشغل المنجز من قبل التوربين W . اما الحرارة المطرودة Q_2 تتمثل في الحرارة التي يسحبها الماء في المكثف. اما الخزان الحار

Boiler

Hot receiver هو المرجل والخزان البارد

Cold receiver هو المكثف والنظام هو البخار.

condenser

مثال ثاني للماكنة الحرارية هو دورة التوربين الهوائي المغلقة يتم تجهيز الحرارة الى الهواء في المسخن Heater عن طريق التماس غير المباشر مع الغازات الساخنة ويتمدد الهواء في التوربين منجزا شغلا w ويبرد في Cooler بواسطة الماء طارحا الحرارة Q_2 الخزان الحار Hot receiver هو Heater والخزان البارد هو Cold receiver هو Cooler والنظام هو الهواء.

Coefficient of Performance :

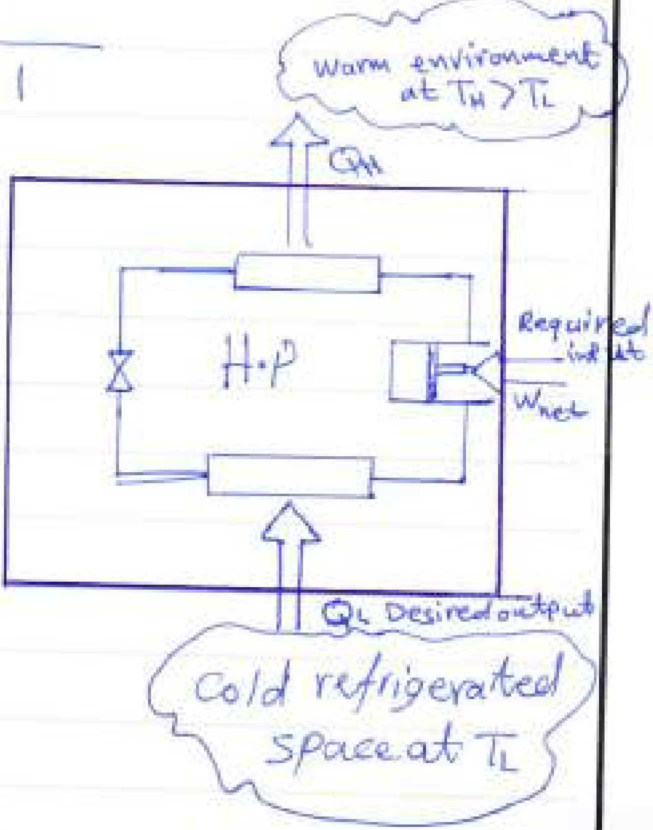
The efficiency of refrigerator is expressed in terms of the coefficient of Performance (CoP), denoted by COP_R. The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this objective, it requires a work input of $W_{net,in}$. Then the COP of a refrigerator can be expressed as

$$COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{Q_L}{W_{net,in}}$$

$$W_{net,in} = Q_H - Q_L$$

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Notice that the value of COP_R can be greater than unity. that is, the amount of heat removed from the refrigerated space can be greater than the amount of work input.

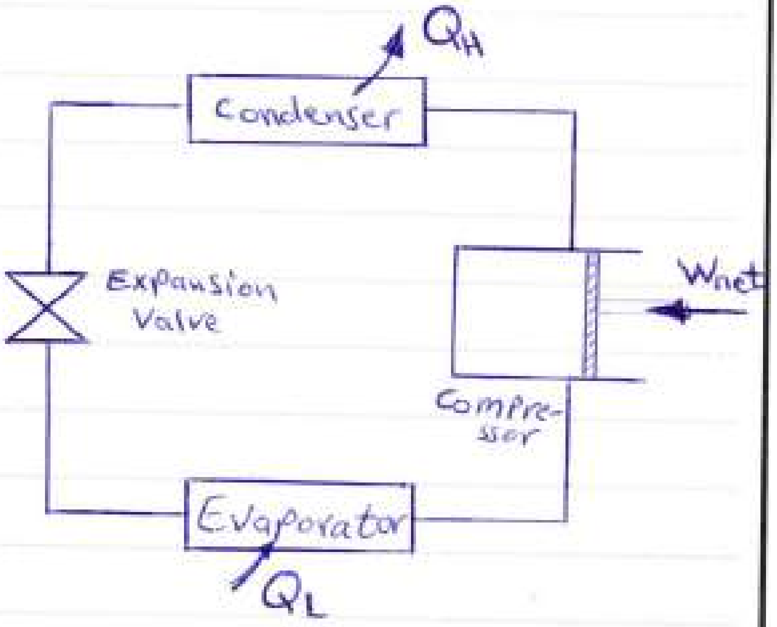


- Refrigerators and Heat Pumps:

We known that the heat flows in the direction of decreasing temperature (from high-temp. media to Low temp.), this heat transfer Process occurs in nature without requiring any devices.

The reverse Process cannot occure by itself. The transfer of heat from a low-temp. medium to a high temp. one requires special devices called refrigerators.

Refrigerators, like heat engines are cyclic devices. The working fluid used in the refrigeration cycle is called a Refrigerant. This cycle which involves four main components: Compressor, a Condenser, an expansion valve, and an evaporator

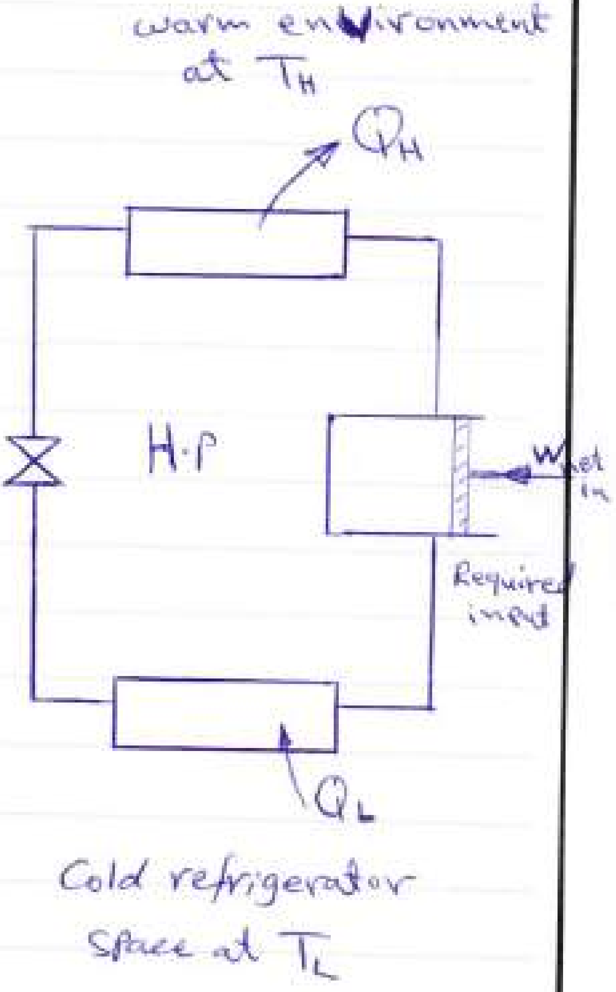


- The Heat Pump:-

It is another device that transfers heat from a low temperature medium to a high-temperature one.

Refrigators and heat pumps operate on the same cycle but differ in their objectives.

The measure of performance of heat pump is also expressed in terms of the Coefficient of performance COP_{HP}.



$$COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{Q_H}{W_{net,in}}$$

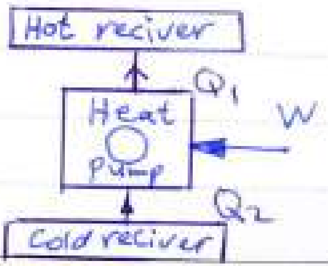
$$COP_{H.P} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

$$Q_H = Q_L + W_{net\,in}$$

$$COP_{HP} = \frac{Q_L + W_{net\,in}}{W_{net,in}}$$

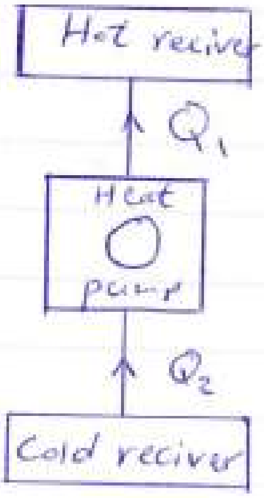
$$= \frac{Q_L}{W_{net\,in}} + 1$$

$$COP_{H.P} = COP_R + 1$$



2. The Clausius statement :-

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower temp. body to a higher temp. body.



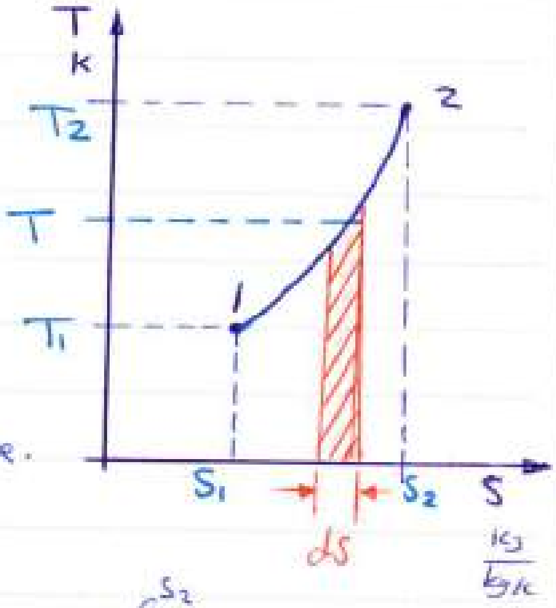
" ENTROPY "

The term entropy which literally means transformation was first introduced by clausius. It is an important thermodynamics Property of a working Substance, which increas with the addition of heat and decreases with its removal.

In a reversible Process, over a Small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working Substance. Mathematically, heat absorbed by the working Substance.

Heat transfer reversibly = Area under the Curve

$= T \cdot ds$



Total heat transfer 1 → 2

$= $ total area under the Curve.

$= \sum_{S=S_1}^{S=S_2} T \cdot ds$

Heat transfer reversibly from 1 → 2 $= \int_{S_1}^{S_2} T \cdot ds = Q_{rev.}$ ---(1)

Differentiating eq(1) $\Rightarrow dQ_{rev.} = T \cdot ds$

$$\therefore \boxed{ds = \frac{dQ_{rev.}}{T}} \quad \text{----}(2)$$

This equation gives the relationship which must exist between S, T and $Q_{rev.}$ in order that the area of the graph shall be heat transferred reversibly.

Now it is Possible to calculate the amount of heat transferred reversibly during a non-flow process. Thus by using equation (2) changes in (S) can be determined. It is this function (S) which is called entropy.

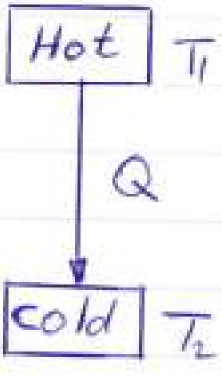
If heat is received which makes $Q_{rev.}$ Positive, then the entropy of the receiving substance has increased.

If heat is rejected, which makes $Q_{rev.}$ negative then the entropy of the rejecting substance is decreased.

Now consider a given quantity of heat energy Q. let this quantity of heat be lost from a hot fluid at temperature T_1 and let it be received by a cooler fluid at temperature T_2.

$$S = \frac{Q}{T} = \frac{Q}{T_1}$$

$$T_1 > T_2 \Rightarrow \frac{Q}{T_2} > \frac{Q}{T_1}$$



so the gain of entropy to the cooler fluid is greater than the loss of entropy from the hot fluid.

– The entropy of vapours (two – phase system)

1. Liquid entropy :

$Q = m c_p (T_2 - T_1)$ for m kg

$Q = c_p (T_2 - T_1)$ for 1 kg

$Q = c_L * \Delta T$ for Liquid

$$\boxed{dQ = c_L * dT} \quad \text{-----} \ (1)$$

c_L = Specific heat capacity of liquid at Constant Pressure
= 4.187 kJ/kg K. for H_2O

Dividing eq. (1) throughout by T, then

$$\frac{dQ}{T} = c_L * \frac{dT}{T} = ds \Rightarrow ds = c_L * \frac{dT}{T}$$

$$\int_{s_1}^{s_2} ds = c_L * \int_{T_1}^{T_2} \frac{dT}{T} \Rightarrow S_2 - S_1 = c_L * \ln \frac{T_2}{T_1}$$

When $T_1 = 0\ °c = 273\ k \Rightarrow S_1 = 0$

$$S_2 - 0 = c_L * \ln \frac{T_2}{273}$$

$$\therefore \boxed{S = c_L * \ln \frac{T}{273}}$$

Ex. 8 Determine the value of the specific entropy of water at 100 °C

Sol.: $S = 4.187 \ln\left(\frac{T}{273}\right)$

$\therefore S_f = 4.187 \ln \frac{373}{273} = 1.31$ kJ/kg K

from steam table at 100 °C $S_f = 1.3068$ kJ/kg K.

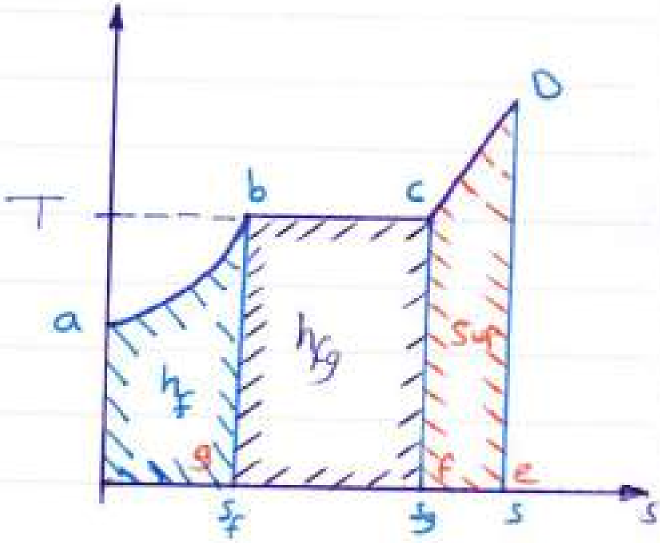
2. Entropy of evaporation:

Heat transferred = Change of enthalpy

Area abgo = h_f

" bcfg = h_{fg}

" cdef = $h_{sup.}$



Area bcfg $= h_{fg} = T_s (S_g - S_f)$

$S_g - S_f = \frac{h_{fg}}{T_s}$ = Specific

entropy of evaporation $= S_{fg}$ for wet steam

$S = S_f + x\, S_{fg}$

$$\boxed{S = C_p \ln \frac{T_s}{273} + x \cdot \frac{h_{fg}}{T_s}}$$ for wet steam

Ex.:- Determine the value of the specific entropy of wet steam at pressure of 2000 kPa and 0.8 dry.

a- by calculation

b- " Using values of entropy from st. table.

Sol.:

a- $S = C_{pL} \cdot \ln \frac{T_s}{273} + X * \frac{h_{fg}}{T_s}$

from steam table at 2000 kPa $T_s = 212.42\ ^{\circ}C$

$h_{fg} = 1890.74$ kJ/kg

$\therefore S = 4.187 * \ln \frac{485.4}{273} + 0.8 * \frac{1890.74}{485.4} = 5.525\ \frac{kJ}{kgK}$

b- $S = S_f + X\, S_{fg}$

at 2000 kPa $\rightarrow S_f = 2.4473$ kJ/kgK

$S_{fg} = 3.8935$ kJ/kgK

$\therefore S = 2.4473 + 0.8 * 3.8935 = 5.562$ kJ/kgK

3. Entropy of Superheated vapour:-

Area cdef $= C_{pv} \cdot dT$

C_{pv} = specific heat capacity of superheated vapour at constant pressure.

$dQ = C_{pv} * dT \quad \div T \Rightarrow \frac{dQ}{T} = C_{pv} \frac{dT}{T} = ds$

$$\int_{S_g}^{S} dS = C_{pv} \cdot \int_{T_s}^{T} \frac{dT}{T} \Rightarrow S - S_g = C_{pv} \times \ln \frac{T}{T_s}$$

$$S = S_g + C_{pv} \ln \frac{T}{T_s}$$

$$\therefore \quad \boxed{S = C_{pL} \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \times \ln \frac{T}{T_s}}$$

Ex. :-

Determine the value of specific entropy of steam at 1500 kPa with a temperature of 300 °C, $C_{pv} = 2.0934 \frac{kJ}{kg\,K}$

a- by calculation

b- from St. tables.

Sol.

a- $$S = C_{pL} \times \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \ln \frac{T}{T_s}$$

from St. tables at 1500 kPa $t_s = 198.32\ °C = 471.32\ K$

$h_{fg} = 1947.28\ kJ/kg$

$$S = 4.187 \times \ln \frac{471.32}{273} + \frac{1947.28}{471.32} + 2.0934 \times \ln \frac{573}{471.32}$$

$= 6.8268\ kJ/kg\,K$.

b- from Superheated Steam tables at 1500 kPa & 300 °C

$$\frac{1500 - 1400}{1600 - 1400} = \frac{S - 6.9533}{6.8844 - 6.9533} \Rightarrow S = 6.9188 \frac{kJ}{kg\,K}$$

- The Entropy of Ideal gas (Single - Phase System) :-

$dQ = dW + dU$ for non - flow Process

$dQ = dw + du$ for unit mass

$dq = du + dw = C_v dT + P dv \div T$ .... (1)

$\frac{dq}{T} = C_v \frac{dT}{T} + \frac{P}{T} dv$ .... (2)

$PV = RT \Rightarrow (P/T) = R/V$ .... (3)

Sub. eq ③ in ②

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v} \Rightarrow \int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} C_v \frac{dT}{T} + \int_{v_1}^{v_2} R \frac{dv}{v}$$

$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}} \quad \text{.....}(5)$$

eq. 5 represent the relation between (S, T, v).

$R = C_p - C_v$

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{v_2}{v_1} \quad \text{....} (6)$$

$$S_2 - S_1 = C_v \left(\ln \frac{T_2}{T_1} - \ln \frac{v_2}{v_1} \right) + C_p \ln \frac{v_2}{v_1} \quad \text{....} (7)$$

$$S_2 - S_1 = C_v \ln \left(\frac{T_2}{T_1} \cdot \frac{v_1}{v_2} \right) + C_p \ln \frac{v_2}{v_1} \quad \text{......} (8)$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \frac{v_1}{v_2}$$

Sub. eq(9) in (8) $\Rightarrow \boxed{S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1}}$ ....(10)

eq. (10) represent the relation between (S, P, V)

$C_v = C_p - R$ Sub. in eq. (5)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{----(5)}$$

$$\therefore \; S_2 - S_1 = (C_p - R) \cdot \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{---(11)}$$

$$= C_p \ln \frac{T_2}{T_1} - R \left(\ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1} \right) \quad \text{---(12)}$$

$$= C_p \cdot \ln \frac{T_2}{T_1} - R \ln \left(\frac{T_2}{T_1} \cdot \frac{V_1}{V_2} \right) \quad \text{---(13)}$$

$$\therefore \; \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$\therefore \; \boxed{S_2 - S_1 = C_p \cdot \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad \text{-----(14)}$$

eq. 14 represent the relation between (S, T, P)

- Isometric Process $V = C$

from eq. (5) & (10)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{---(5)}$$

$V_2 = V_1 \Rightarrow (V_2 / V_1) = 1 \Rightarrow \ln 1 = 0$

$$\therefore \; S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$$

and $S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1}$ ----(10)

$v_2 = v_1 \Rightarrow \ln(v_2/v_1) = 0$

$\therefore S_2 - S_1 = C_v \ln \frac{P_2}{P_1}$

$\therefore \boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} = C_v \ln \frac{P_2}{P_1}}$ for Constant Volume

- Isobaric Process $P = C$

from eq. (10) & (14)

$S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1}$ ----(10)

$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ -----(14)

But $P_1 = P_2 \Rightarrow \ln\left(\frac{P_2}{P_1}\right) = \ln(1) = 0$

$\therefore \boxed{S_2 - S_1 = C_p \ln \frac{v_2}{v_1} = C_p \ln \frac{T_2}{T_1}}$ for Constant pressure

- Isothermal Process $T = C$

from eq. (5) & 14

$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ ---(5)

$T_2 = T_1 \Rightarrow \ln(T_2/T_1) = \ln(1) = 0$

$\therefore S_2 - S_1 = R \ln \frac{v_2}{v_1}$

and $S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ ----(14)

also $\ln(T_2/T_1) = 0 \Rightarrow S_2 - S_1 = -R \ln \frac{P_2}{P_1} = R \ln \frac{P_1}{P_2}$

$\therefore \boxed{S_2 - S_1 = R \ln \frac{v_2}{v_1} = R \ln \frac{P_1}{P_2}}$ for const. Temp.

- Polytropic Process $PV^n = C$

$Q = \frac{\gamma - n}{\gamma - 1} * W$

$dQ = \frac{\gamma - n}{\gamma - 1} P dv \div T \Rightarrow \frac{dQ}{T} = ds = \frac{\gamma - n}{\gamma - 1} \frac{P dv}{T}$

$Pv = RT \Rightarrow \frac{P}{T} = \frac{R}{v} \Rightarrow ds = \frac{\gamma - n}{\gamma - 1} \cdot R \frac{dv}{v}$

$\int_{S_1}^{S_2} ds = \frac{\gamma - n}{\gamma - 1} R \int_{v_1}^{v_2} \frac{dv}{v} = S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} R \ln \frac{v_2}{v_1}$

$C_v = \frac{R}{\gamma - 1} \Rightarrow R = C_v(\gamma - 1)$

$S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} * C_v(\gamma - 1) \ln \frac{v_2}{v_1}$

$\boxed{S_2 - S_1 = C_v(\gamma - n) \ln \frac{v_2}{v_1}}$; $\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$

$\boxed{S_2 - S_1 = C_v \frac{\gamma - n}{n - 1} \ln \frac{T_1}{T_2}}$; $\frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$

$\boxed{S_2 - S_1 = C_v \frac{\gamma - n}{n} \ln \frac{P_1}{P_2}}$

and

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{P_2}{P_1} = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

- Isentropic Process $S = C$

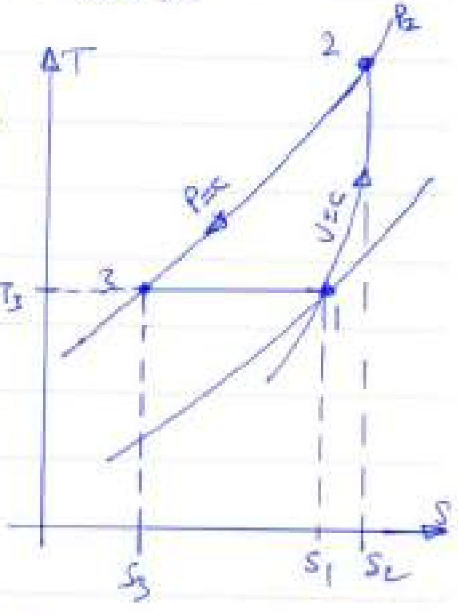
$$S_2 - S_1 = 0$$

Ex.: Air at 15c and 1.05 bar occupies 0.02 m³. The air is heated at constant volume until the pressure is 4.2 bar and then cooled at constant pressure back to the original temp. Calculate the heat flow to or from the air and the net entropy change. Sketch the processes on T-S diagram.

Sol.:- $R = 0.287$ kJ/kg.k , $C_v = 0.717$ kJ/kgk , $C_p = 1.005$ kJ/kg k.

$P_1 V_1 = m R T_1 \Rightarrow 1.05 * 10^2 * 0.02 = m * 0.287 * 288 \Rightarrow m = 0.0254$ kg

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, $V = C \Rightarrow T_2 = \frac{P_2 * T_1}{P_1} = \frac{4.2 * 288}{1.05} = 1152$ k



$Q_{1\to2} = m C_v (T_2 - T_1) = 0.0254 * 0.717 * (1152 - 288)$
$= 15.75$ kJ

$Q_{2\to3} = m C_p (T_3 - T_2) = 0.0254 * 1.005 (288 - 1152)$
$= -22.05$ kJ

$Q_{3\to1} = Q_{1\to2} + Q_{2\to3} = 15.75 - 22.05$
$= -6.3$ kJ

for $V = C$, $S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$ kJ/kg.k

or $S_2 - S_1 = m\, C_v \ln T_2/T_1$ kJ/k

$= 0.0254 * 0.717 \ln (1152/288) = 0.0253$ kJ/k

for $P = C$ from $2 \to 3$ $S_3 - S_2 = 0.0254 * 1.005 \ln \frac{288}{1152} = -0.0354$ kJ/k

$\Delta S_{net} = \Delta S_{1\to2} + \Delta S_{2\to3} = 0.0253 - 0.0352 \simeq -0.0101$ kJ/k

The Heat Engine Cycles :-

A system is said to have undergone a cycle if it returns to it's initial state at the end of the process, and we have two types of cycles 1. Thermodynamic cycles. 2- Mechanical cycles.

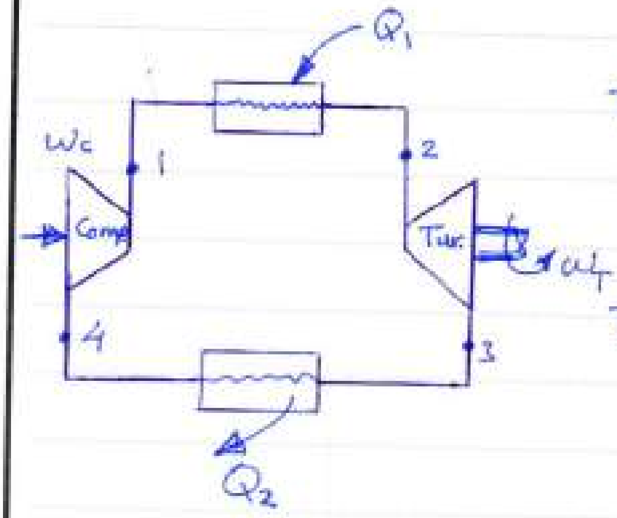
It can be shown that there is an ideal theoretical cycle which is the most efficient conceivable; this cycle is called the **Carnot** cycle. The highest thermal efficiency possible for a heat engine in practice is only about half that of the ideal theoretical Carnot cycle, between the same temperature limits.

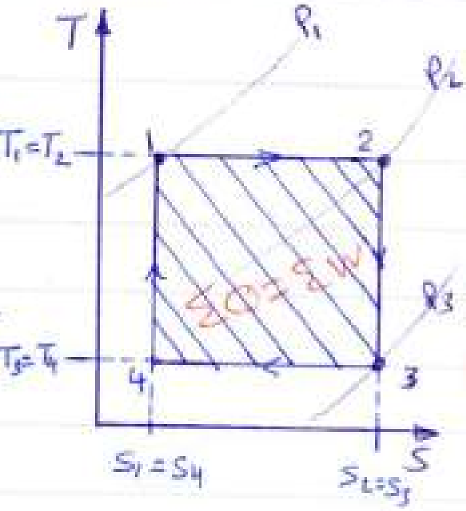
The Carnot cycle :-

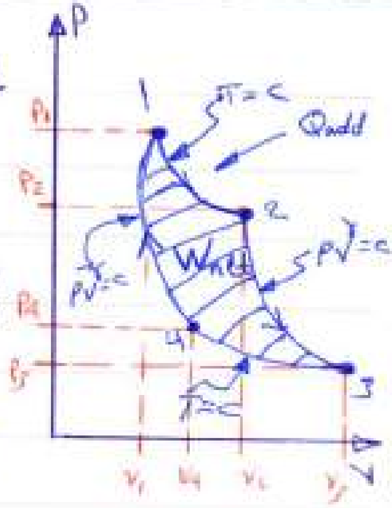
It can be shown from the Second law of thermodynamics that no heat engine can be more efficient than a reversible heat engine working between the same temperature.

Carnot, a French engineer, showed that the most efficient possible cycle is one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature.

The cycle therefore consists of two isothermaly processes joined by two adiabatic process. Since all the process are reversible, then the adiabatic processes in the cycle are also isentropic.


Q1
Wc
1
2
Comp
Tur.
Wt
4
3
Q2


T
P1
P2
P3
T1=T2
T3=T4
1
2
3
4
S1=S4
S2=S3
S


P
P1
P2
P4
P3
1
2
3
4
T=c
Qadd
PV^γ=c
Wnet
PV^γ=c
T=c
V1
V4
V2
V3
V

Notes:

1. Since the heat absorbed is at the highest temperature and rejected at the Lowest temperature, the Carnot cycle would give a maximum possible efficiency.

2. It may be noted that it is impossible to make a steam engine working on Carnot cycle, the isothermal expansion $4 \rightarrow 1$ and the isothermal Compression $2 \rightarrow 3$ will have to be Carriedout extermely slow to ensure that the Steam is always at constant temperature. But the isentropic expansion $1 \rightarrow 2$ and isentropic Compression $3 \rightarrow 4$ should by Carriedout as quickly as possible in order to approach ideal isentropic Condition. we know that sudden changes in the speed of an engine are not possible in actual Practice. therefore it is impossible to realise Carnot's engine in actual practice.

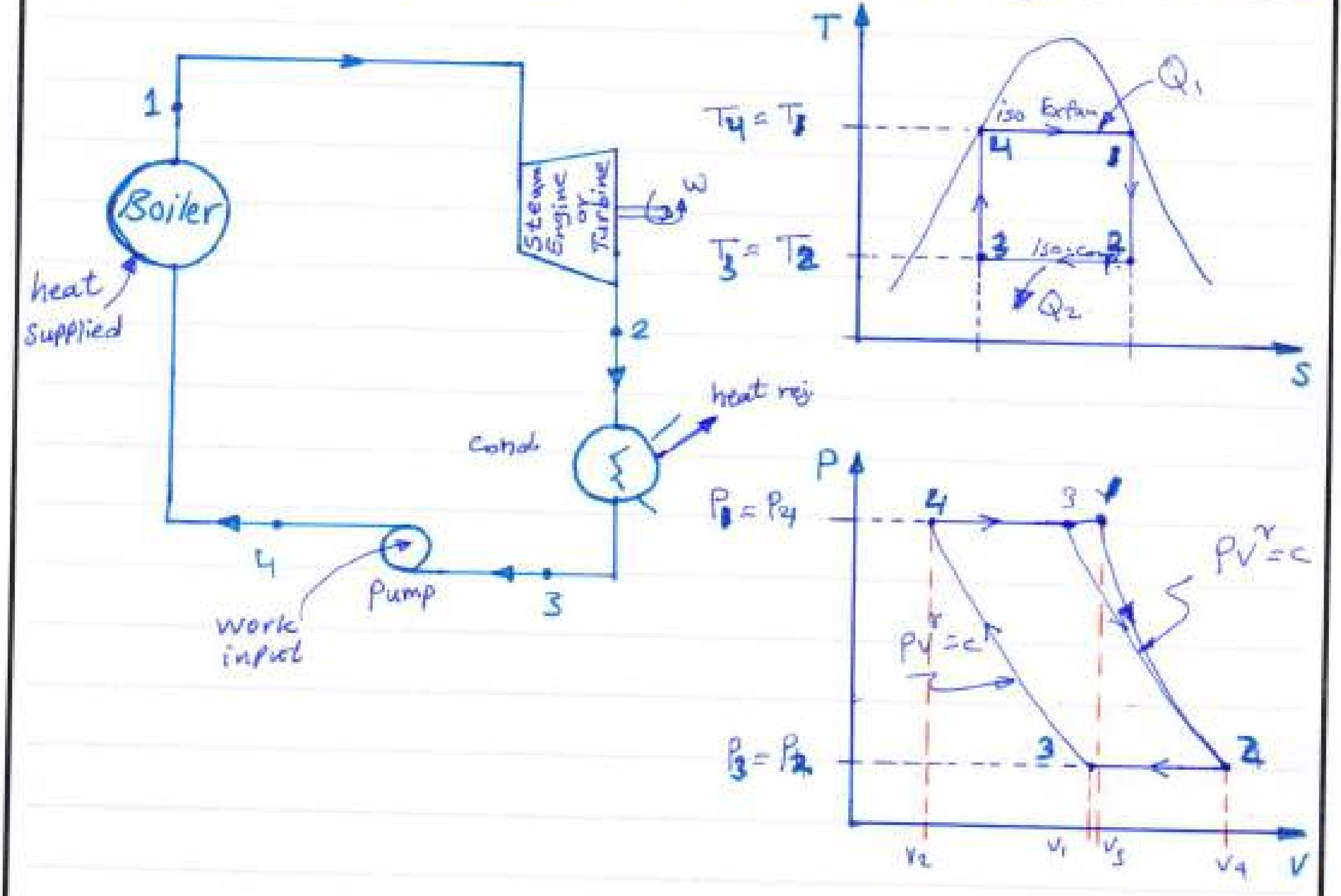
3. At State 3 the steam is wet at T_2, it is difficult to stop Condensation at the Point 3 and then compress it just to state 4 It is more convenient to allow the condensation process to proceed to completion.

4. The working fluid is water at the new state, and this can be conveniently pumped to boiler pressure, the pump has much smaller dimensions than it would have if it had to pump a wet vapour, the compression process is carriedout more efficiently. and the equipment required is simpler and less expensive.

The Steam Cycles :-

In a Vapour cycle, all the theory of air cycles remains the Same, except the working Substance, which is Steam. The Steam may be in any form, wet, dry saturated or superheated.

- Carnot Cycle with steam as working substance:-



Process 1 → 2

The dry steam now expands isentropically in a steam engine or turbine. the pressure and temperature fails from P_1 to P_2 and T_1 to T_2 respectively. and no heat is supplied or rejected during this process.

Process 2 → 3

The wet steam is now isothermally condensed in condenser and heat is rejected at constant temperature and pressure.

$$q_{2\to3} = T_2 * (S_2 - S_3) = T_3 * (S_2 - S_3)$$

$$\sum Q = \sum W$$

∴ work done during the cycle = Heat absorbed − Heat rejected.

$$W.D = Q_1 - Q_2 \qquad \text{for 1 kg}$$

$$W.D = q_{4\to1} - q_{2\to3}$$

$$W.D = T_1 * (S_1 - S_4) - T_2 (S_2 - S_3)$$

$$S_1 = S_2 \quad , \quad S_3 = S_4 \quad ; \quad T_1 = T_4 \, , \quad T_2 = T_3$$

$$\therefore W.D = (T_1 - T_2) * (S_1 - S_4)$$

or

$$= (T_1 - T_2)(S_2 - S_3)$$

or $$= (T_4 - T_3)(S_1 - S_2)$$

or $$= (T_4 - T_3)(S_2 - S_3)$$

Process: 3 → 4

Isentropic Compression of wet Steam in Pump the Pressure and temperature rises, and no heat is absorbed or rejected during this Process. $Q=0$

Process 4 → 1

The Saturated water at Point (4) is isothermally Converted into dry Saturated Steam, in a boiler and the heat is absorbed at Constant temperature.

$q_{4\to1}$ = Change in entropy * Absolute temperature.

$$q_{4\to1} = T_1 * (S_1 - S_4) = T_4 * (S_1 - S_4)$$

$$\eta = \frac{\text{work done}}{\text{heat added}} = \frac{(T_1 - T_2)(S_1 - S_4)}{T_1 (S_1 - S_4)} = \frac{T_1 - T_2}{T_1}$$

$$\boxed{\eta_{th} = 1 - \frac{T_2}{T_1}}$$

Ex: A Power Plant is supplied with dry saturated steam at a pressure of 16 bars and exhausts at 0.2 bar, find the efficiency of the Carnot cycle.

Sol.

from Steam table at 16 bar, $t_s = 201.4\ ^\circ c$

$\therefore T_1 = 201.4 + 273 = 474.4\ K$

from Steam table at 0.2 bar, $t_s = 60.1\ ^\circ c$

$\therefore T_2 = 333.1\ k$

$$\eta_{th} = 1 - \frac{T_2}{T_1} = 1 - \frac{333.1}{474.4} = 0.298 \text{ or } 29.8\%$$

Ex.: In a Carnot cycle, heat is supplied at 350 °c and is rejected at 25 °c, The working fluid is water, which while receiving heat, evaporates from liquid at 350 °c to steam at 350 °c from steam tables the entropy change for this Process is 1.438 kJ/kg k

Sol.: the heat Supplied $= T_1 * (S_1 - S_4)$; $T_1 = 350 + 273 = 623\ k$

$= 1.438 * 623 = 895.87\ kJ/kg$

the work done $= (T_1 - T_2) * (S_1 - S_4)$

$= 1.438 * (632 - 298) = 467.35\ \frac{kJ}{kg}$

the heat rejected $= T_2 (S_2 - S_3)$

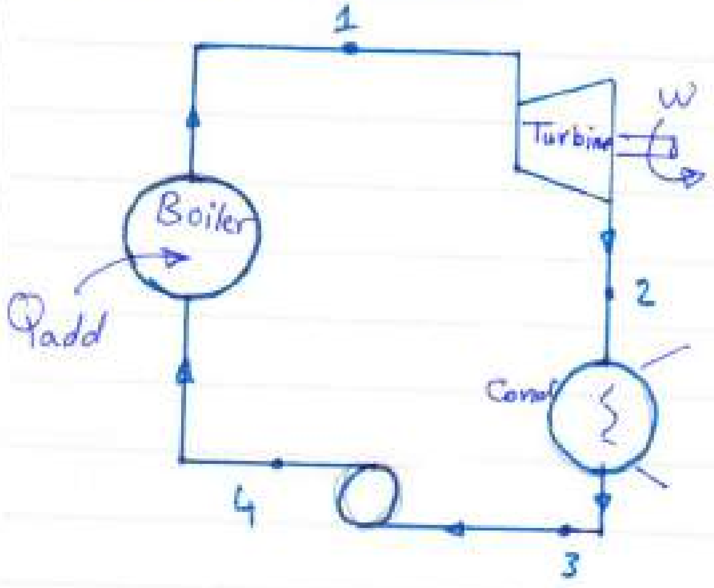
$= 1.438 * 298 = 428.52\ kJ/kg$

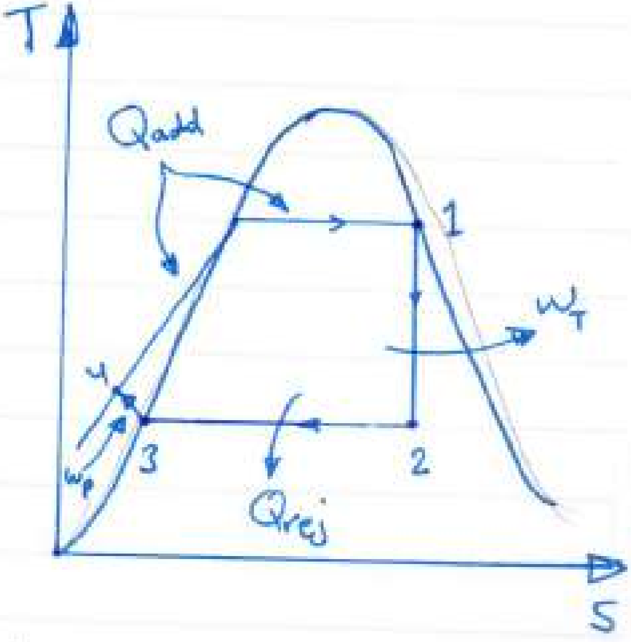
$T_2 = 25 + 273$

$= 298\ k.$

- Rankine Cycle :-

The Rankine cycle is an ideal cycle for Comparing the Performance of Steam Plants. It is modified form of Carnot cycle, in which the Condensation Process (2 → 3) is continued until the steam is Condensed into water.





By the flow energy equation. $\Delta Z = 0$ $\Delta C = 0$

1. Boiler 4 → 1 Heat added Q_1

$h_4 + q = h_1 + W$, $W = 0$ in the boiler

$\therefore q = h_1 - h_4$

$$\Rightarrow \boxed{Q_{4\to 1} = m(h_1 - h_4)}$$

2. Turbine $1 \rightarrow 2$ (Work output, W_t)

$$h_1 + q = h_2 + w \quad ; \quad q = 0 \text{ , isentropic expansion.}$$

$$\therefore W_{1\to2} = h_1 - h_2 \Rightarrow \boxed{W = m(h_1 - h_2)}$$

3. Condenser $2 \rightarrow 3$ (Heat rejected) Q_2

$$h_2 + q = h_3 + w \quad , \quad w = 0$$

$$\Rightarrow q = h_3 - h_2$$

for positive value $q = h_2 - h_3$

$$\boxed{Q_{2\to3} = m(h_2 - h_3)}$$

4. Pump $3 \rightarrow 4$ (Work input, W_p) ; $q = 0$

$$h_3 + q = h_4 + w \quad ; \quad q = 0$$

$$\Rightarrow W_{3\to4} = h_3 - h_4$$ $\Rightarrow$ for Positive value

$$W_{3\to4} = h_4 - h_3 \Rightarrow \boxed{W_p = m(h_4 - h_3)}$$

Or $W_{Pump} = v_f(P_1 - P_2)$

Net work done $= W_{1\to2} - W_{3\to4} = W_T - W_P$

$$W.D = (h_1 - h_2) - (h_4 - h_3)$$

If the feed Pump work is neglected, $W_P = 0$

$$W.D = h_1 - h_2$$

Heat added $= h_1 - h_4$

Rankine efficiency, $\eta_R = \dfrac{W.D}{Q_{add}} = \dfrac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_4)}$

Work ratio: It is defined as the ratio of net work output to the gross output.

$$\text{Work ratio} \; \frac{\text{Net work output}}{\text{Gross output}} = \frac{\text{Turbine work} - \text{Pump work}}{\text{Turbine work}}$$

It may be noted that the Carnot cycle, despit of its high ideal thermal efficiency, has low work ratio. It is one of the reasons that Carnot cycle is not attempted. The higher value of work ratio also means a smaller size of the plant.

* Specific Steam Consumption (S.S.C.):

It is also known as steam rate or specific rate of flow of steam. It is defined as the mass of steam that must be supplied to a steam engine or turbine in order to develop a unit amount of work or power output.

The amount of work or power output is usually expressed in kilowatt hour (kwh).

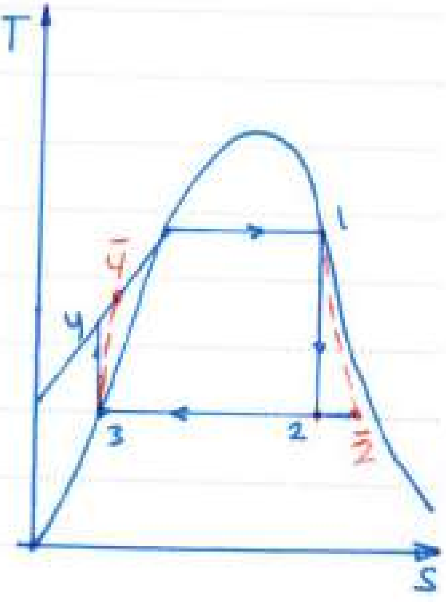
$$S.S.C. = \frac{1\,kwh}{W} = \frac{3600}{w} = \frac{3600}{h_1 - h_2} \quad . kg/kw.h \,,\; w_p = 0$$

w = net work done.

* Isentropic efficiency :-

a- For expansion process : (Turbine 1→2)

$$\eta_{ise} = \frac{\text{actual } W.D}{\text{Isentropic } W.D} = \frac{h_1 - h_{\bar{2}}}{h_1 - h_2}$$



b- for Compression process (Pump)

$$\eta_{ise} = \frac{\text{Isentropic } W.D}{\text{actual } W.D} = \frac{h_4 - h_3}{h_{4}^{\circ} - h_3}$$

Ex.: A steam power plant operates between a boiler pressure of 42 bars and a condenser pressure of 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and specific steam consumption:

a- for a Carnot cycle using wet steam.

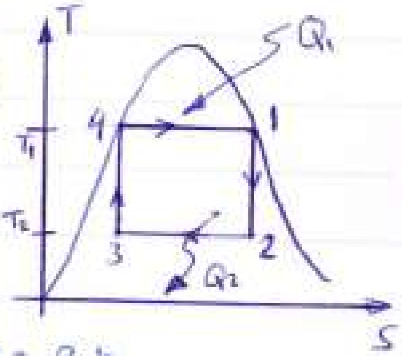
b- for a Rankine cycle with dry saturated steam at entry to the turbine.

c- for the " " of (b) when the expansion process has an isentropic efficiency of 80%

Sol.:

for steam table at $P = 4200\ kPa$ → $t_s = 253.2\ °C$

at $P = 3.5\ kPa$ → $t_s = 26.7\ °C$



a- $\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{267+273}{253.2+273} = 43.2\%$

$Q_1 = h_1 - h_4 = h_{g_1} - h_{f_4}$ at $P = 4200\ kPa$

$\therefore Q = 2800 - 1105 = 1695\ kJ/kg$

$Net\ work = \eta_{Car.} * Q_1 = 0.432 * 1695 = 732.3\ kJ/kg$

$W_T = h_1 - h_2$; $1 \rightarrow 2$ $\Delta S = 0 \Rightarrow S_1 = S_2 = S_g$ at $4200\ kPa$

$\therefore S_2 = 6.05\ kJ/kg\,K$

$S_2 = S_{f_2} + X_2 S_{fg_2}$ at $= 3.5\ kPa$

$6.05 = 0.391 + X_2 * 8.13 \Rightarrow X_2 = 0.696$

$\therefore h_2 = h_{f_2} + X_2 * h_{fg_2}$ at $3.5\ kPa$

$h_2 = 112 + 0.696 * 2438 = 1808\ kJ/kg$

$W_T = h_1 - h_2 = 2800 - 1808 = 992\ kJ/kg$

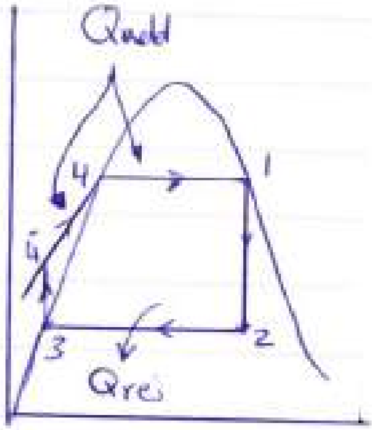
$Work\ ratio = \frac{net\ work}{gross\ work} = \frac{732.3}{992} = 0.738$

$$S.S.C = \frac{3600}{W_{net}} = \frac{3600}{732.3} = 4.92 \text{ kg/kwh.}$$

b- $h_1 = 2800$ kJ/kg . $h_2 = 1808$ kJ/kg ; $h_4 = 1105$ kJ/kg

$h_3 = 111.9$ kJ/kg

$v_3 = v_f = 0.001 \text{ m}^3/\text{kg}$



$$\eta_R = \frac{W.D}{Q_1} = \frac{(h_1 - h_2) - (h_{\bar{4}} - h_3)}{(h_1 - h_{\bar{4}})}$$

$$\text{pump work} = h_{\bar{4}} - h_3 = v_3 (P_1 - P_2)$$
$$= 0.001 \times (42 - 0.075) \times 10^3$$
$$= 4.2 \text{ kJ/kg}$$

$$h_{\bar{4}} = 4.2 + h_3 = 4.2 + 111.9 = 116.1 \text{ kJ/kg}$$

$W_T = h_1 - h_2 = 2800 - 1808 = 992$ kJ/kg ; $W_p = 4.2$ kJ/kg

$Q_1 = h_1 - h_{\bar{4}} = 2800 - 116.1 = 2684$ kJ/kg

$$\text{work ratio} = \frac{W_{net}}{\text{gross work}} = \frac{992 - 4.2}{992} = 0.995$$

$S.S.C. = 3600/W_{net} = 3600/(992 - 4.2) = 3.64$ kg/kwh

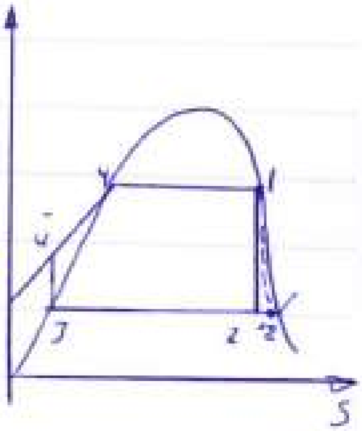
$$\eta_R = \frac{W.D}{Q_1} = \frac{992 - 4.2}{2684} = 0.368 \text{ or } 36.8\%$$

c-

$$\eta_{ise} = \frac{h_1 - h_{\bar{2}}}{h_1 - h_2} = 0.8$$

$0.8 * (2800 - 1808) = 2800 - h_{\bar{2}} \Rightarrow h_{\bar{2}} = 2006.4 \frac{\text{kJ}}{\text{kg}}$

$$\eta_R = \frac{(h_1 - h_{\bar{2}}) - (h_{\bar{4}} - h_3)}{(h_1 - h_2)} = \frac{(2800 - 2006.4) - 4.2}{2800 - 116.1}$$
$$= 0.294$$

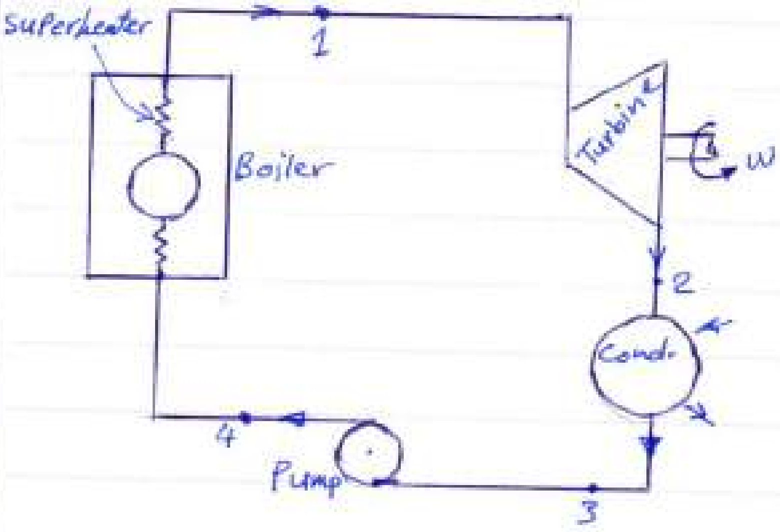


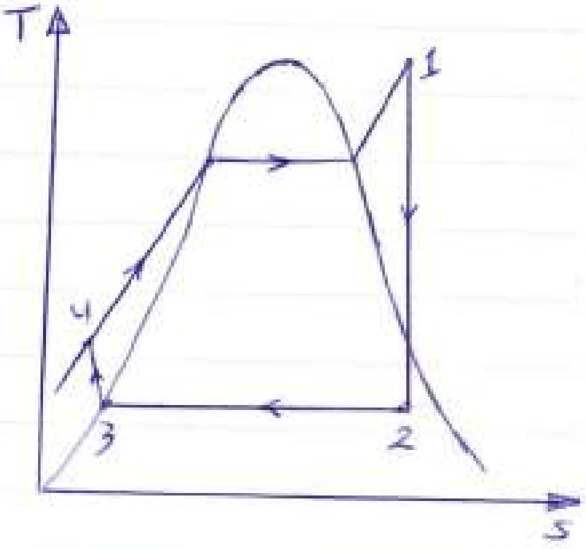
$$\text{Work ratio} = \frac{(2800 - 2006.4) - 4.2}{2800 - 2006.4} = 0.9947$$

$$\text{S.S.C.} = \frac{3600}{793.6 - 4.2} = 4.56 \text{ Kg/Kwh.}$$

- Rankine Cycle with Superheated Steam:

The average temperature at which heat is supplied in the boiler can be increased by superheating the steam. Usually the dry saturated steam from the boiler drum is passed through a second bank of smaller bore tubes within the boiler. This bank is situated such that it is heated by hot gases from the furnace until the steam reaches the required temperature.





$$Q_1 = Q_{add} = h_1 - h_4$$

$$W.D = W_T - W_P = (h_1 - h_2) - (h_4 - h_3)$$

$$\Rightarrow \boxed{\eta_{th} = \frac{W.D}{Q_{add}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}}$$

Ex. : A steam turbine operating on the Rankine cycle received steam from the boiler at 3.5 MPa and 350 °C and exhaust to the condenser at 10 kPa. The condensate is then returned to the boiler by the feed pump. Calculate, neglecting all losses:

a. the Energy supplied in the boiler per kg of steam.

b. the dryness fraction of steam entering the condenser.

c. the Rankine efficiency.

Sol.

at 3500 kPa & 350 °C $\Rightarrow h_1 = 3104 \frac{kJ}{kg}$; $S_1 = 6.6579 \frac{kJ}{kg\,K}$

$\Delta S = 0 \quad S_1 = S_2 = 6.6579$ kJ/kgK

$S_2 = S_{f_2} + x_2 S_{fg_2}$ at 10 kPa $\Rightarrow 6.6579 = 0.6493 + X_2 * 7.5$

$\Rightarrow X_2 = 0.8$; $h_2 = h_{f_2} + x_2 h_{fg_2}$ at 10 kPa

$\therefore h_2 = 191.8 + 0.8 * 2392.8 = 2106$ kJ/kg

$h_4 = h_f$ at 10 kPa $\Rightarrow h_4 = 192$ kJ/kg , $w_p = 0$

Now Energy Supplied $= h_1 - h_4 = 3014 - 192 = 2912.2$ kJ/kg.

$$\eta_R = \frac{W.D}{Q_{add}} = \frac{3104 - 2106}{3104 - 191.8} = 0.3427 \text{ or } 34.27\ \%$$

Ex. : Compare the Rankine cycle performance of example in page (116) with that obtained when the steam is superheated to 500, neglecting wp.

Sol. : from steam table at 4200 kPa & 500 °C $\Rightarrow h_1 = 3442.6$ kJ/kg

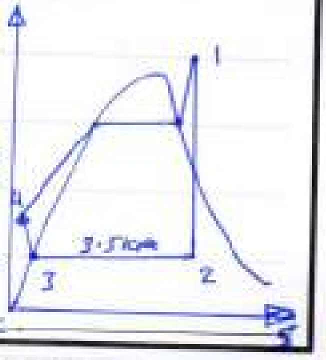
$S_1 = 7.066$ kJ/kgK ; $\Delta S = 0 \Rightarrow S_1 = S_2 = 7.066 = S_{f_2} + X_2 S_{fg_2}$ at 3.5 kPa

$7.066 = 0.391 + X_2 * 8.13 \Rightarrow X_2 = 0.821 \Rightarrow h_2 = 112 + 0.821 * 2438$

$h_2 = 2113$ kJ/kg. ; $h_3 = h_f = 112$ kJ/kg $\therefore w_T = h_1 - h_2$

$W_T = 3442.6 - 2113 = 1329.6$ kJ/kg ; $Q_{add} = h_1 - h_3 = 3442.6 - 112 = 3330.6 \frac{kJ}{kg}$

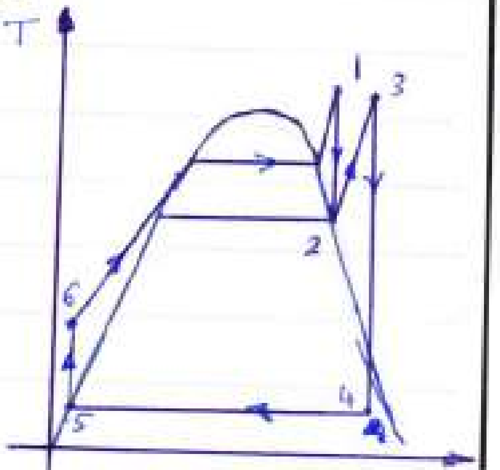
$$\eta_R = \frac{h_1 - h_2}{h_1 - h_3} = \frac{1329.6}{3330.6} = 0.399 \text{ ; } S.S.C = \frac{3600}{h_1 - h_2} = \frac{3600}{1329.6} = 2.71 \text{ kg/kWh}$$



The η_R increased due to superheating and improvement S.S.C.

- Reheat Steam Cycle :-

It is desirable to increase the average temperature at which heat is supplied to the Steam, and also to keep the steam as dry as possible in the lower pressure stages of the turbine. The wetness at exhaust Should be no greater then 10%. The high boiler pressure are required for high efficiency, but the expansion in one stage can result in exhaust Steam which is wet. This is a condition which is improved by superheating the Steam. The exhaust Steam condition Can be improved must effectively by reheating the Steam, the expansion being Carriedout in two Stages.



Heat Supplied $= Q_{6\to1} + Q_{2\to3}$

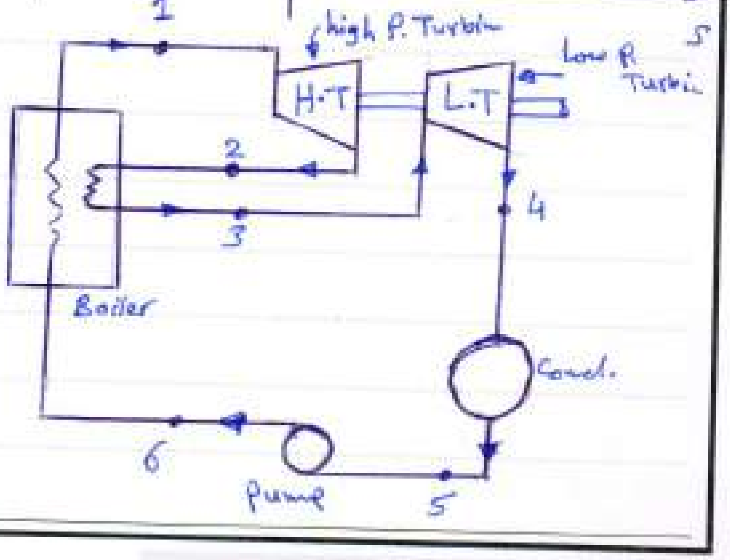
$Q_{6\to1} = h_1 - h_6$

$Q_{2\to3} = h_3 - h_2$

Work output $= W_{1\to2} + W_{3\to4}$

$W_{1\to2} = h_1 - h_2$; $W_{3\to4} = h_3 - h_4$; if $W_p = 0$

$$\eta_{th} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_6) + (h_3 - h_2)}$$



Ex.: The Steam Condition at inlet to the turbine are 42 bars and 500 °c and Condenser Pressure is 0.035 bar. Assume that the Steam is just dry Saturated on leaving the first turbine, and reheat to the initial temperature. Neglect feed water work. Calculate the Cycle efficiency.

Sol.:-

Condition 1: $P_1 = 4200$ kpa ; $T_1 = 500$ °c

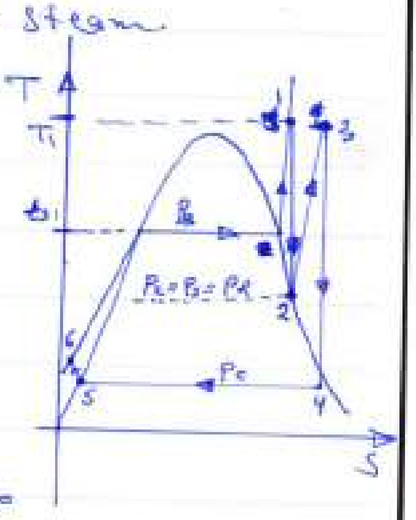
at 4200 kPa $\Rightarrow t_{s_1} = ?$

$$\frac{4200 - 4000}{5000 - 4000} = \frac{t_{s_1} - 250.4}{263.99 - 250.4} \Rightarrow t_{s_1} = 253.1 \text{ °c}$$

$\because T_1 > t_{s_1} \Rightarrow$ The State is superheated steam.

From steam table at 4200 kPa + 500 °c

By Interpolation:-



$$\frac{4200 - 4000}{5000 - 4000} = \frac{h_1 - 3445.21}{3433.76 - 3445.21} = \frac{S_1 - 7.09}{6.9758 - 7.09}$$

$h_1 = 3442.92 \frac{kJ}{kg}$, $S_1 = 7.067$ kJ/kg.k

Condition (2): $S_1 = S_2 = S_{g_2} = 7.067$ kJ/kg.k also

at $S_{g_2} = 7.067$ kJ/kg.K $\Rightarrow P_2 = ?$

$$\Rightarrow \frac{7.067 - 7.0526}{7.0878 - 7.0526} = \frac{P_2 - 250}{225 - 250} \Rightarrow P_2 = 239.7 \text{ kpa}$$

$$\frac{7.067 - 7.0526}{7.0878 - 7.0526} = \frac{h_{g2} - 2716.89}{2712.04 - 2716.89} \Rightarrow h_2 = h_{g_2} = 2714.9 \text{ kJ/kg}$$

condition (3) ; $P_3 = P_2 = P_R = 239.7$ kPa $\Rightarrow$ $T_3 = 500$ °c is given at $P_3 = 239.7$ kPa and 500 c $\Rightarrow h_3 = ?$ the State is superheat

By Interpolation;-

$$\frac{239.7-200}{300-200} = \frac{h_3 - 3487.03}{3485.96-3487.03} \Rightarrow \frac{S_3 - 8.5132}{8.3250-8.5132}$$

$\Rightarrow h_3 = 3486.6$ kJ/kg ; $S_3 = 8.438$ kJ/kg .

Condition (4):- $P_4 = P_{cond.} = 0.035$ bar $= 3.5$ kPa

$\Delta S = 0 \Rightarrow S_3 = S_4 = 8.438$ kJ/kg.K

at 3.5 kPa $\Rightarrow S_{g_4} = ? \Rightarrow \frac{3.5-3}{4-3} = \frac{S_{g4} - 8.5775}{8.4746 - 8.5775}$

$\therefore S_{g_4} = 8.526$ kJ/kg.k $\Rightarrow \because S_{g_4} > S_4 \Rightarrow$ the State is wet st.

$\therefore S_4 = S_{f_4} + X S_{fg_4}$ at 3.5 kPa $\Rightarrow$ ~~[illegible]~~

$$\frac{3.5-3}{4-3} = \frac{S_{f_4} - 0.3545}{0.4226-0.3545} = \frac{S_{fg_4} - 8.2231}{8.052-8.2231} \Rightarrow S_{f_4} = 0.3885 \ \frac{kJ}{kg\,k}$$

$S_{fg_4} = 8.137$ kJ/kg.k

$\therefore 8.438 = 0.3885 + X_4 * (8.137)$

$\Rightarrow X_4 = 0.98$

$$0.5 = \frac{h_{f_4} - 101.03}{121.44-101.03} = \frac{h_{fg_4} - 2444.47}{2432.93-2444.47} \Rightarrow \begin{array}{l} h_{f_4} = 111.23 \text{ kJ/kg} \\ h_{fg_4} = 2438.7 \ \frac{kJ}{kg} \end{array}$$

$h_4 = 111.23 + 0.98 * (2438.7) = 2501.156$ kJ/kg.

Condition (5):- $P_5 = P_{cond} = 3.5$ kPa the state is Liquid sat.

$h_5 = h_f$ at 3.5 kPa $\Rightarrow h_5 = 111.23 \ \frac{kJ}{kg}$ $S_5 = 0.3885$ kJ/kg k

Condition (6) . neglecting work pump. is given $\Rightarrow h_6 = h_5 = 111.23$ kJ/kg

Now.

$$Q_{add} = Q_1 + Q_2 = (h_1 - h_6) + (h_3 - h_2)$$

$$= (3442.92 - 111.23) + (3486.6 - 2714.9)$$

$$\therefore Q_{add} = 4103.39 \text{ kJ/kg}$$

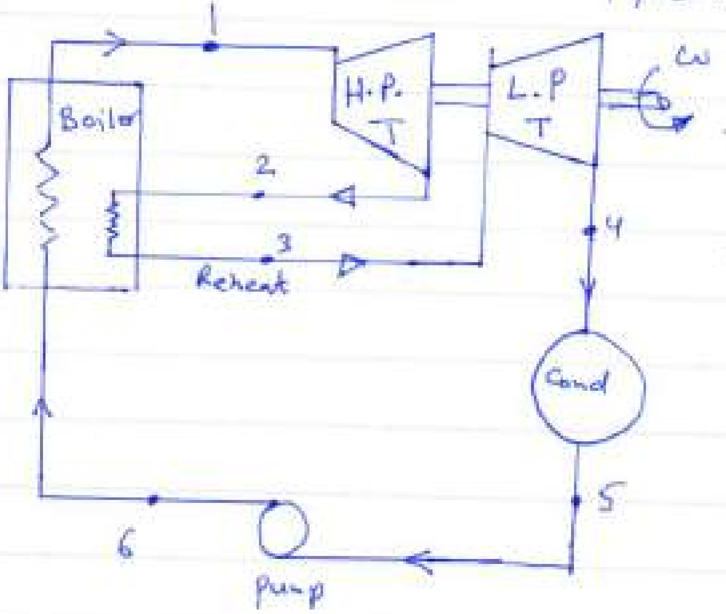
$$W_T = W_1 + W_2 = (h_1 - h_2) + (h_3 - h_4)$$

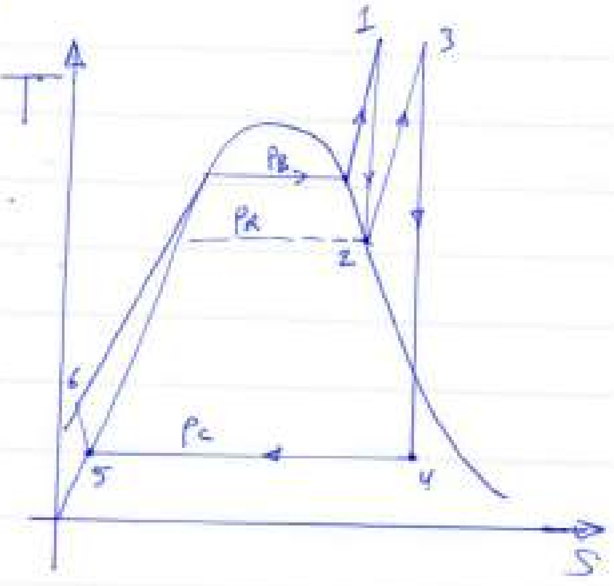
$$= (3442.92 - 2714.9) + (3486.6 - 2501.156)$$

$$\therefore W_T = 1713.464 \text{ kJ/kg k}$$

$$\eta_{th} = \frac{W_{net}}{Q_{add}} = \frac{1713.464}{4103.39} = 0.417 \text{ or } 41.7\ \%$$

$$S.S.C = \frac{3600}{W_{net}} = \frac{3600}{1713.464} = 2.1 \text{ kg/kw.h}$$





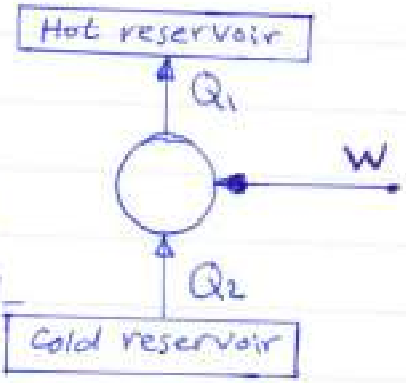
" Vapour Compression Refrigeration System "

Refrigeration in the engineering sense, means maintaining a system at a temperature less than the temperature of the surroundings. This will not occur naturally, so a device must be developed that will maintain this condition.

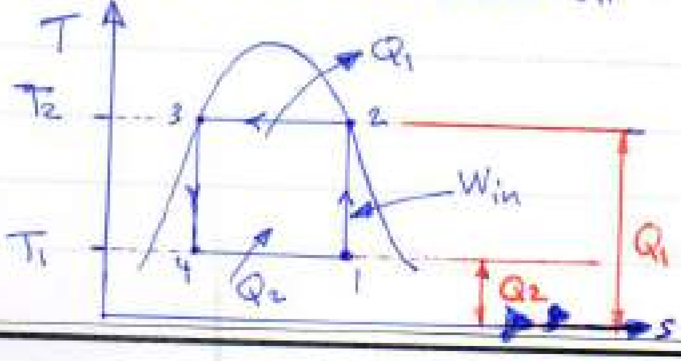
The vapour compression refrigeration system is now used for all purpose refrigeration. It is generally used for all industrial purposes from a small domestic refrigerator to a big air conditioning plant.

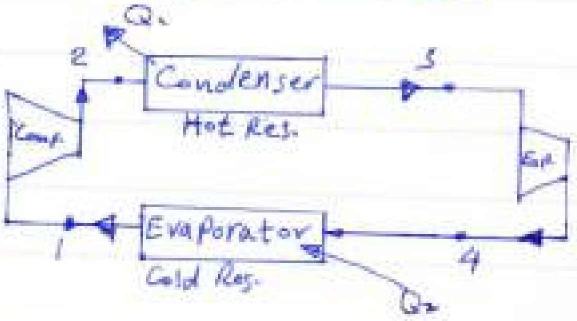
A reversed Carnot engine will removed heat from a low temperature reservoir and deliver this energy, plus work necessary to transfer the heat, to high temperature reservoir. The refrigerated system in this case is the low temperature reservoir.

– Reversed Carnot Cycle :–



In refrigeration Carnot cycle the heat is absorbed from the cold reservoir and rejected to the hot reservoir.





- Processes of Refrigeration Carnot cycle :-

1 → 2 : Work input to the compressor to compress to dry saturated vapour, rising its pressure and temperature to dry or superheated condition.

2 → 3 : heat rejected in condenser from the vapour to the ambient, changing the dry, or superheated vapour to saturated liquid.

3 → 4 : throttling the saturated liquid (expansion) changing it to wet vapour.

4 → 1 : heat absorbed from cold reservoir, changing the wet vapour of law quality to wet vapour of high quality.

Work input (Comp. $1 \rightarrow 2$) $= Q_1 - Q_2$

Heat rejected (Process $2 \rightarrow 3$) $= T_2 * (S_2 - S_3) = Q_1$

Heat absorbed (Process $4 \rightarrow 1$) $= T_1 * (S_1 - S_4) = Q_2$

Since $S_3 = S_4$ & $S_2 = S_1 \Rightarrow Q_2 = T_1 * (S_2 - S_3)$.

Coefficient of Performance (CoP) :-

The Performance ratio of refrigeration system is not the efficiency, but rather the Coefficient of Performance, and define as the refrigeration effect (heat absorbed) divided by the net work done on the cycle (work input).

$$CoP_R = \frac{Q_2}{W} = \frac{T_1 \cdot (S_2 - S_3)}{T_2 (S_2 - S_3) - T_1 (S_2 - S_3)} = \frac{T_1}{T_2 - T_1}$$

$$CoP_R = \frac{T_{Low}}{T_{high} - T_{Low}}$$

It is more suitable to change the names of the processes of the reversed Carnot Cycle to:

Heat absorbed to Refrigeration effect $Q_2 = T_1 * (S_2 - S_3)$
Heat rejected to Heat rejected in the Condenser $Q_1 = T_2 (S_2 - S_3)$
work input to work input to compressor $W = Q_1 - Q_2$

Notes :-

tonne of refrigeration (TR)

$1\ TR = 1000 * 335\ kJ$ in 24 hours.

$$= \frac{1000 * 335}{60 * 24} = 232.6\ kJ/min$$

Ex.: A refrigerator has working temperature in the evaporator and condenser of $-30\,^\circ C$ and $32\,^\circ C$ respectively, what is the maximum COP, [illegible] ?, If the actual COP of 0.75 of the maximum COP, Calculate the refrigeration effect in kW per kW of power plant.

Sol.

$$COP = \frac{T_1}{T_2 - T_1} = \frac{-30+273}{(32+273)-(-30+273)} = 3.91$$

$$\text{actual } COP_r = 0.75 * 3.91 = 2.939$$

$$COP_r = \frac{Q_2}{W} \Rightarrow 2.939 = \frac{Q_2}{1}$$

$$\therefore Q_2 = 2.939 \text{ kW of refrigeration / kW of work input}$$

Ex.:

A machine working on a Carnot cycle operates between 305 K and 260 K. Determine the COP, when it is operated as 1. a refrigerating machine. 2. a heat pump. 3- a heat Engine.

Sol.

1. for refrigerating machine

$$COP_R = T_1/(T_2 - T_1) = 260/(305-260) = 5.78$$

2- for heat pump

$$COP_P = T_2/(T_2 - T_1) = 305/(305-260) = 6.78$$

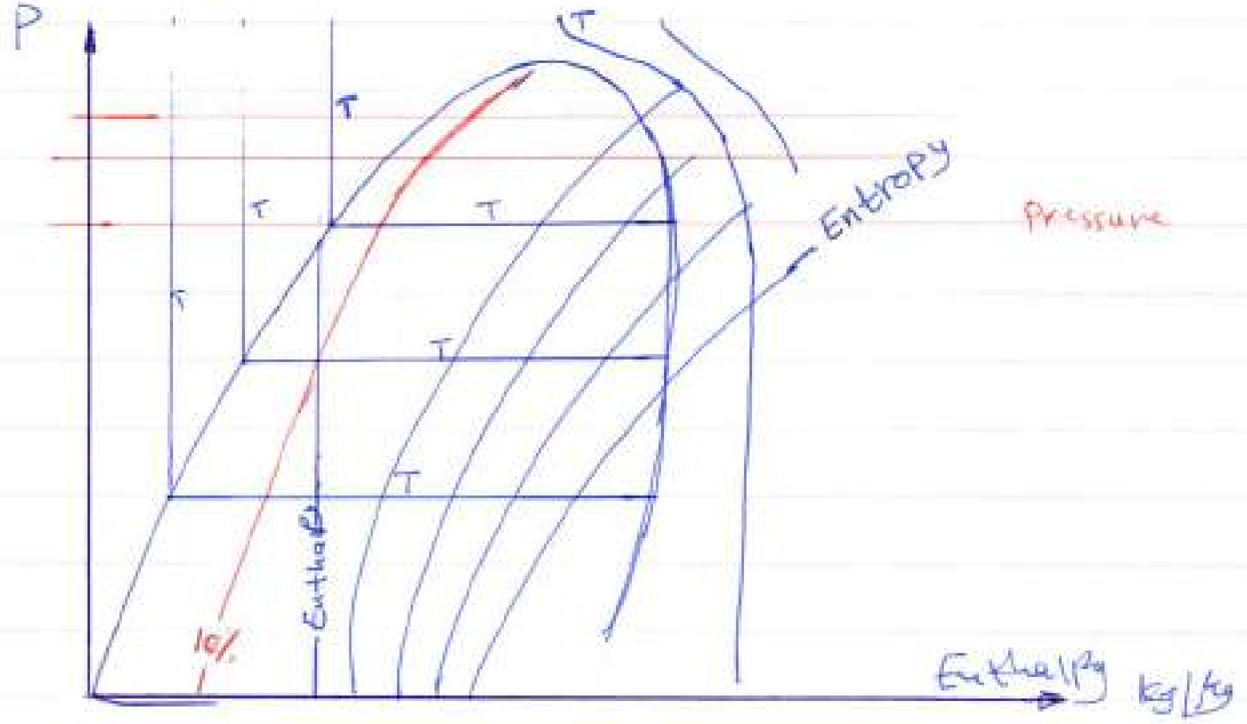
$$\text{or } COP_P = COP_R + 1 = 5.78 + 1 = 6.78$$

3- for Heat Engine

$$COP_E = (T_2 - T_1)/T_2 = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{260}{305} = 0.147$$

- Pressure- Enthalpy (P-h) Chart:

The most convenient chart for studing the behavior of a refrigerant is the p-h chart in which the vertical ordinate represent pressure and horizontal ordinates represent enthalpy.

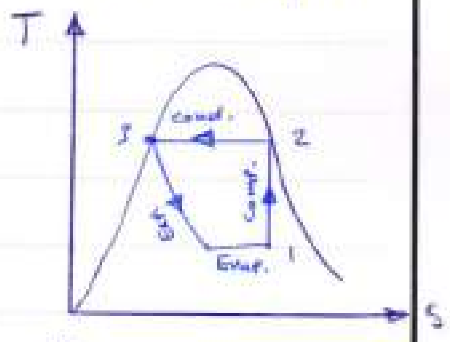


- Types of Vapour Compression Cycle:-

We have already discussed that a vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles. yet the following are important from the subject Point of View.

1. cycle with dry saturated vapour after Compression,
2. " " wet vapour after compression.
3. " " superheated vapour after Compression.
4. " " " " before "
5. " " undercooling or Subcooling of refrigerant.

1- Vapour Compression cycle with dry saturated vapour after Compression.



Processes: 1 → 2 Compression process

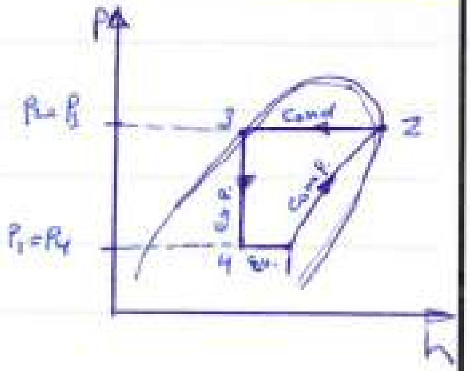
$W_{1\to 2} = h_2 - h_1$ kJ/kg

2 → 3 : Condensing Process

heat rejected $= Q_1 = h_2 - h_3$

$T_2 = T_3$; $P_2 = P_3$

$Q_1 = T_2 * (S_2 - S_3) = T_2 * S_{fg}$



3 → 4 : Expansion Process (throttling process)

$h_3 = h_4$; $h_3 = h_f$ at $P_{cond.}$

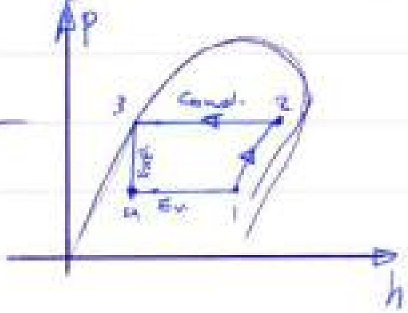
for Expansion Process using Expansion Valve or a Capillary tube

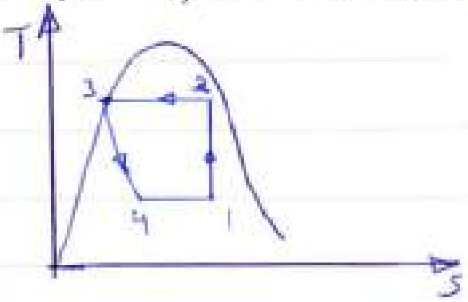
4 → 1 : Vaporising Process

$= h_1 - h_4$; $h_3 = h_4$; $COP = \frac{\text{Refrigerating Effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1}$

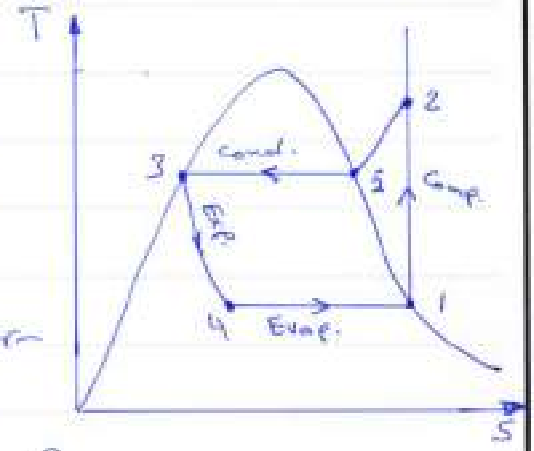
2. Vapour Compression Cycle with wet Vapour after Compression

$$CoP = \frac{h_1 - h_3}{h_2 - h_1}$$



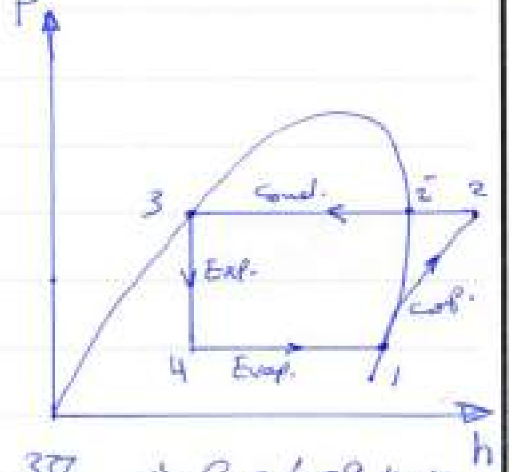


3. Vapour Compression Cycle with Superheated Vapour after Compression :-



Ex. :- A vapour compression refrigerator uses methyl chloride (R40) and operating between temperature limits of −10 °C and 45 °C. At entry to the Compressor, the refrigerant is dry saturated and after compression it ac at temperature of 60 °C. Find the CoP of refrigerator.

Tsat. (°C)	h_f	h_g	s_f	s_g
−10	45.4	460.7	0.183	1.637
45	133	483.6	0.485	1.587



Sol. :

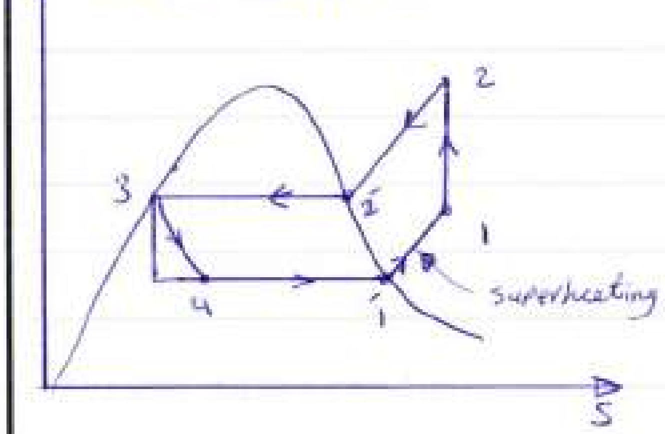
$$s_2 - s_{\bar{2}} = C_p \ln \frac{T_2}{T_{\bar{2}}}$$

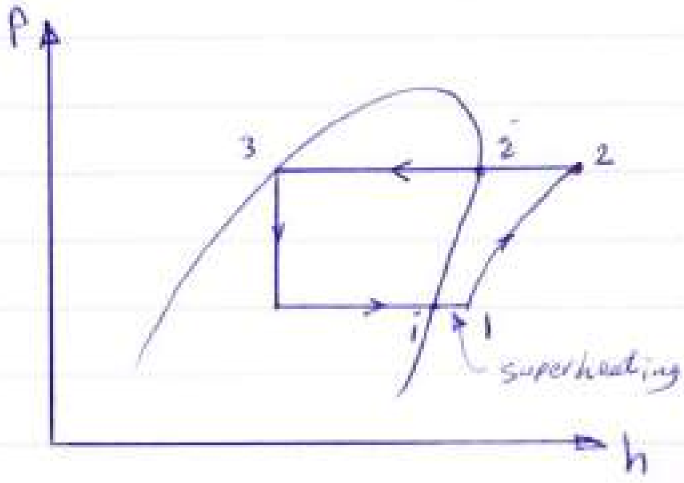
$$s_2 = s_{\bar{2}} + C_p \ln \frac{T_2}{T_{\bar{2}}} = s_1 = 1.637 = 1.587 + C_p \ln \frac{333}{318} \Rightarrow C_p = 1.09 \ kJ/kg\,K$$

$$h_2 = h_{\bar{2}} + C_p * \text{Degree of superheater} = 483.6 + 1.09(333-318) = 500 \ \frac{kJ}{kg}$$

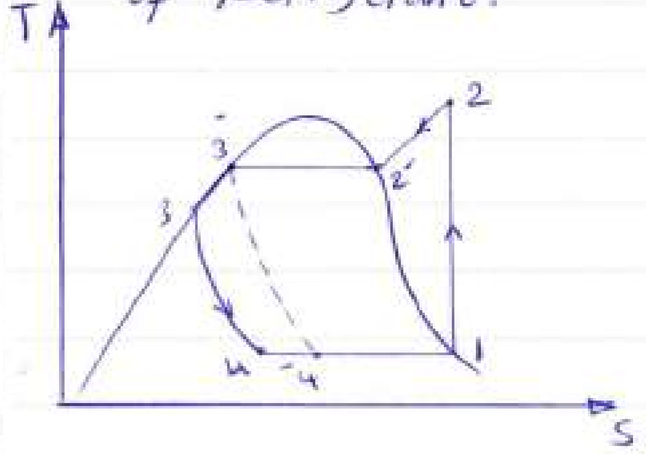
$$\therefore \; COP = \frac{h_1 - h_f}{h_2 - h_1} = \frac{460.7 - 133}{500 - 460.7} = 3.77$$

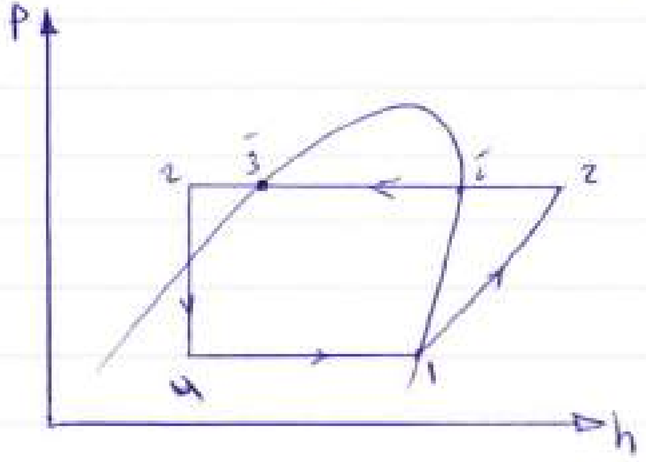
4. Vapour Compression cycle with Superheated vapour before Compression :-





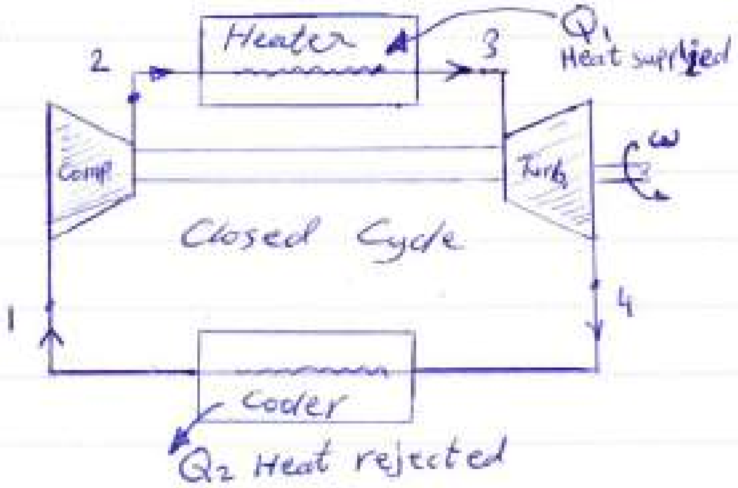
5. Vapour Compression Cycle with Undercooling or Subcooling of Refrigerant.

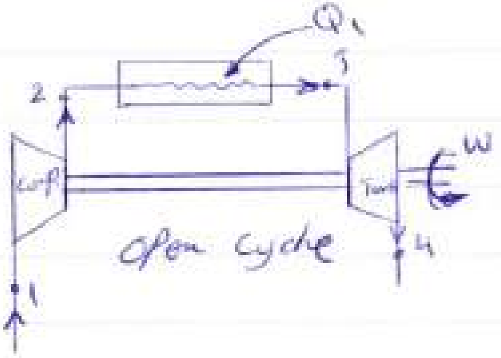




- Joule or Brayton Cycle :- « Constant Pressure cycle »

In this cycle heat supply and heat rejection processes occur reversibly at constant pressure. The expansion and compression processes are isentropic. This cycle was at one time used as the ideal basis for a hot-air reciprocating engine.





- Process : 1 → 2 (adiabatic reversible compression) $PV^{\gamma} = c$

$$gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w$$

$z_1 \simeq z_2$, $C_1 \simeq C_2$, $q = 0$

∴ Work input to compressor $= h_1 - h_2 = C_p(T_1 - T_2)$

- Process: 2 → 3 « constant pressure heat addition » $P = c$

$$gz_2 + h_2 + \frac{1}{2}C_2^2 + q = gz_3 + h_3 + \frac{1}{2}C_3^2 + w$$

$z_2 \simeq z_3$, $C_2 \simeq C_3$, $w = 0$

∴ heat supplied to heater $= h_3 - h_2 = C_p(T_3 - T_2)$

- Process : 3 → 4 « adiabatic reversible expansion » $PV^{\gamma} = c$

$$gz_3 + h_3 + \frac{1}{2}C_3^2 + q = gz_4 + h_4 + \frac{1}{2}C_4^2 + w \; ; z_3 = z_4 , C_3 = C_4 , q = 0$$

∴ Work output from the turbine $= h_3 - h_4 = C_p(T_3 - T_4)$

Process: 4 → 1 " Constant Pressure heat rejected " $P = c$

$$g z_4 + h_4 + \frac{1}{2} C_4^2 + q_r = g z_1 + h_1 + \frac{1}{2} C_1^2 + W$$

$$z_4 \simeq z_1 \quad , \quad C_4 = C_1 \ . \quad W = 0$$

∴ heat rejected in cooler $= h_1 - h_4 = C_p (T_1 - T_4)$

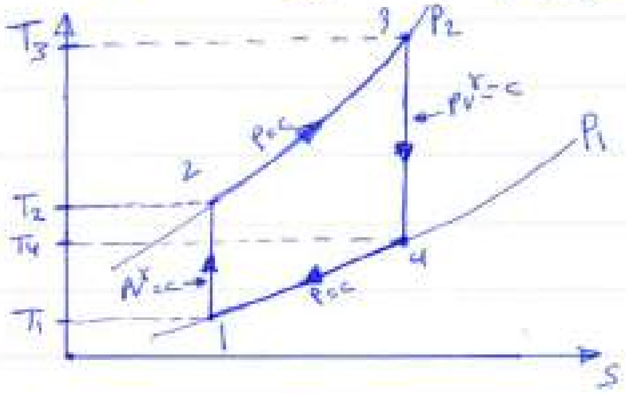
for positive value

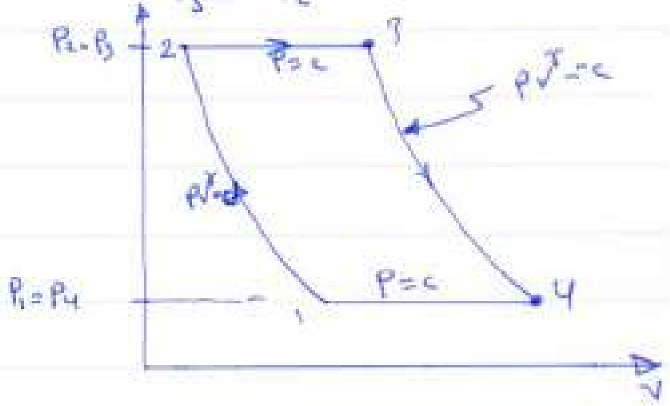
Work Comp. $= h_2 - h_1 = C_p (T_2 - T_1)$

heat rejected $= h_4 - h_1 = C_p (T_4 - T_1)$

$$\text{thermal efficiency} = \frac{\text{net work}}{\text{heat added}} = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad ---- *$$





$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

∵ $P_1 = P_4$ & $P_2 = P_3$ $\Rightarrow$ $\frac{P_2}{P_1} = \frac{P_3}{P_4}$ $\Rightarrow$ ∴ $\frac{T_3}{T_4} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$T_3 = T_4 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$; let $\frac{P_2}{P_1} = r_p$ = Pressure ratio

∴ $T_3 = T_4 \times r_p^{\frac{\gamma-1}{\gamma}}$ --- (1) ; $T_2 = T_1 \times r_p^{\frac{\gamma-1}{\gamma}}$ ---- (2)

Sub eq. 1 & 2 in *eq.

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{T_4 * r_p^{\gamma-1/\gamma} - T_1 * r_p^{\gamma-1/\gamma}}$$

$$\eta_{th} = 1 - \frac{\cancel{(T_4 - T_1)}}{\cancel{(T_4 - T_1)}\, r_p^{\gamma-1/\gamma}} \Rightarrow \boxed{\eta_{th} = 1 - \frac{1}{r_p^{\gamma-1/\gamma}}}$$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{W_{Turb.} - W_{comp.}}{W_T}$$

$$W.R = 1 - \frac{W_{comp.}}{W_T} = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)} =$$

$$= 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$T_2 = T_1 * r_p^{\gamma-1/\gamma} , \quad T_4 = T_3 * \frac{1}{r_p^{\gamma-1/\gamma}}$$

$$W.R = 1 - \frac{T_1 * r_p^{\gamma-1/\gamma} - T_1}{T_3 - T_3 * \frac{1}{r_p^{\gamma-1/\gamma}}} = \frac{T_1 (r_p^{\gamma-1/\gamma} - 1)}{T_3 \left(1 - \frac{1}{r_p^{\gamma-1/\gamma}}\right)}$$

$$W.R = 1 - \frac{T_1 \cancel{(r_p^{\gamma-1/\gamma} - 1)}}{T_3 \frac{\cancel{r_p^{\gamma-1/\gamma} - 1}}{r_p^{\gamma-1/\gamma}}} = 1 - \frac{T_1}{T_3} * r_p^{\frac{\gamma-1}{\gamma}}$$

Ex.: In a gas turbine unit air is drawn in at 1.02 bar and 15 °c and compressed to 6.12 bar. calculate the thermal efficiency and work ratio of the ideal constant pressure cycle with the maximum temperature is 800 °c.

Sol.

$$r_p = \frac{P_2}{P_1} = \frac{6.12}{1.02} = 6 \Rightarrow \eta_{th} = 1 - \frac{1}{r_p^{\gamma-1/\gamma}} = 1 - \frac{1}{6^{\frac{0.4}{1.4}}} = 0.4$$

or 40%

$T_{min.} = T_1 = 15$ °c ; $T_{max} = T_3 = 800$ °c

$$\text{Work ratio} = 1 - \frac{T_1}{T_3} \times r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow w.R = 0.552$$

or

$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$$
$$= 1.005(1073 - 643) - 1.005(481 - 288) = 238 \frac{kJ}{kg}$$

$$W_{gross} = C_p(T_3 - T_4) = 1.005(1073 - 643) = 432 \text{ kJ/kg}$$

$$w.r = \frac{W_{net}}{W_{gross}} = \frac{238}{432} = 0.552.$$

$T_1 = 15 + 273 = 288$ k , $T_3 = 800 + 273 = 1073$ k

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 6^{\frac{0.4}{1.4}} \Rightarrow T_2 = 481 \text{ K}$$

$$\frac{T_3}{T_4} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 643 \text{ k}.$$

Ex. : In an air-Standard Brayton cycle the air enters the Compressor at 0.1 MPa, 15 °C. The Pressure leaving the Compressor is 1.0 MPa, and the maximum temperature in the cycle is 1100 °C. Determine: 1- The Pressure and temp. at each Point in the cycle. 2- The Compressor work, turbine work, heat added and η_{th}:

Sol. :

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = (15+273) * \left(\frac{1}{0.1}\right)^{\frac{0.4}{1.4}} = 556\ K$$

$$W_{comp.} = h_2 - h_1 = C_p(T_2 - T_1) = 1.005\,(556 - 288) = 269.3\ \frac{kJ}{kg}$$

$$W_T = h_3 - h_4 = C_p(T_3 - T_4)\ ;\ T_3 = 1100\ °C = 1373\ K$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 10^{\frac{0.4}{1.4}} \Rightarrow T_4 = 710.8\ K$$

$$W_T = 1.005\,(1373 - 710.8) = 665.5\ kJ/kg$$

$$W_{net} = W_T - W_{comp.} = 665.5 - 269.3 = 396.1\ kJ/kg$$

$$q_{add} = h_3 - h_2 = C_p(T_3 - T_2) = 1.005\,(1373 - 556) = 821\ kJ/kg$$

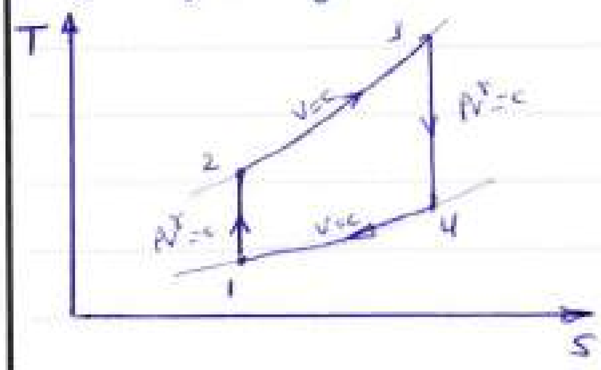
$$\eta_{th} = \frac{W_{net}}{q_{add}} = \frac{396.1}{821} = 0.482 \text{ or } 48.2\ \%$$

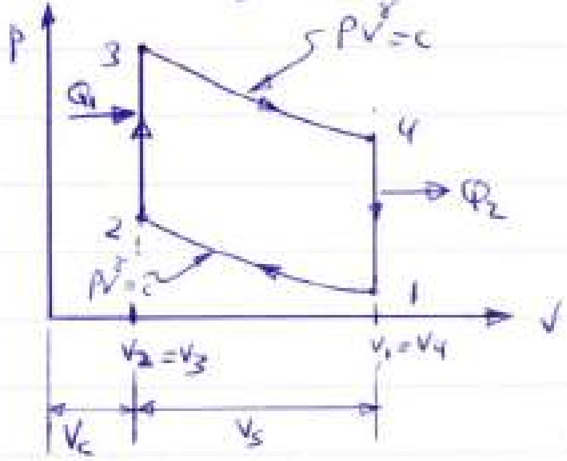
or

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{10^{0.286}} = 48.2\ \%$$

- otto Cycle :- « Constant Volume Cycle »

The otto cycle is the ideal air standard cycle for the Petrol engine gas engine, and the high - speed oil engine.





V_c = clerance volume

V_s = Swept volume

Process: 1→2 Isentropic Compression « Q=0)

$Q = W + \Delta U \;,\; \Rightarrow Q = 0 \Rightarrow W = -\Delta U = C_v(T_1 - T_2)$

Process 2→3 Reversible Constant Volume heat added

$Q = W + \Delta U \;,\; W = 0 \Rightarrow Q = \Delta U = C_v(T_3 - T_2)$

Process 3→4 Isentropic Expansion

$Q = W + \Delta U \;;\; Q = 0 \Rightarrow W = -\Delta U = C_v(T_3 - T_4)$

Process 4→1 Reversible Constant Volume heat rejection

$Q = W + \Delta U \;,\; W = 0 \Rightarrow Q = \Delta U = C_v(T_1 - T_4)$

for Positive Value:

$W_{comp.} = C_v(T_2 - T_1)$

$Q_{rej} = C_v(T_4 - T_1)$

$$r_v = \frac{V_1}{V_2} = \frac{\text{Swept Volume} + \text{Clearance Volume}}{\text{Clearance Volume}}$$

$$\eta_{th} = \frac{\text{Net work}}{\text{heat added}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{c_v (T_4 - T_1)}{c_v (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_3}{T_4} = r_v^{\gamma-1}$$

$$\therefore\ T_3 = T_4 * r_v^{\gamma-1} \quad \& \quad T_2 = T_1 * r_v^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_4 * r_v^{\gamma-1} - T_1 r_v^{\gamma-1}} = 1 - \frac{T_4 - T_1}{(T_4 - T_1)\, r_v^{\gamma-1}}$$

$$\therefore\ \boxed{\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}}$$

Ex.: Calculate the ideal air standard thermal efficiency of Otto cycle for a petrol engine with a cylinder bore of 50mm and a strock of 75 mm and clearance volume of 21.3 cm³

Sol.:

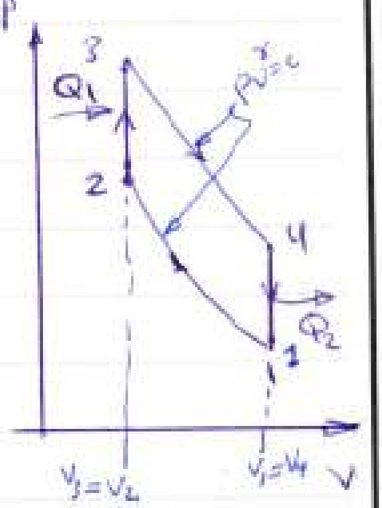
Swept Volume = area of bore * stroke

$$\therefore\ S.V = \frac{\pi}{4} * (5)^2 * 7.5 = 147.26 \text{ cm}^3$$

$$r_v = \frac{147.26 + 21.3}{21.3} = 7.914$$

$$\therefore\ \eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} = 0.56$$

Ex.: In an air Standard Otto Cycle the maximum and minimum temperature are 1400 °C and 15 °C. The heat supplied per kg of air is 800 kJ. Calculate the Compression ratio and the thermal efficiency. Calculate also the ratio of maximum to ~~the~~ minimum pressure in the cycle.



Sol.

$$Q_1 = m C_v (T_3 - T_2) \quad , \quad T_3 = 1400\,^{\circ}C = 1673\,K$$

$$800 = 1 \times 0.717 \times (1673 - T_2) \Rightarrow T_2 = 557.2\,K$$

$$\frac{T_2}{T_1} = r_v^{\gamma-1} \Rightarrow r_v^{0.4} = \frac{557.2}{288} \Rightarrow r_v = 5.2$$

$$\eta_{th} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{5.2^{0.4}} = 0.483$$

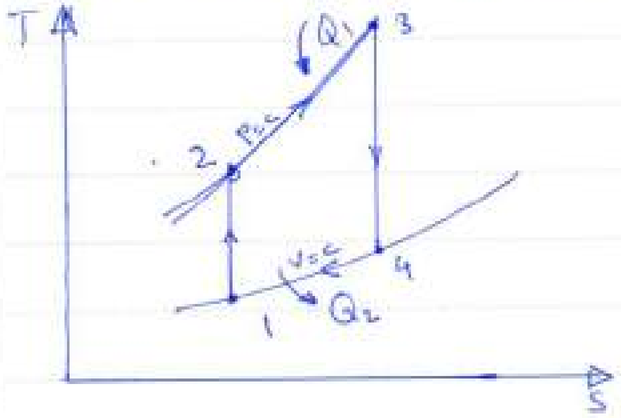
$$\frac{P_3}{P_2} = ?$$

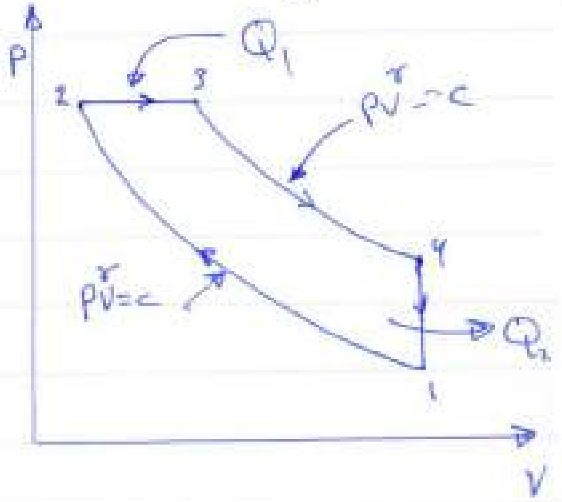
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{P_2}{P_1} = \left(\frac{557.2}{288}\right)^{\frac{1.4}{0.4}} =$$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow \frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow \frac{T_3}{T_2} = \frac{P_3}{P_2} = \frac{1673}{557.2} =$$

- Diesel Cycle:-

This cycle is an important on which all the diesel engines work. It is also known as Constant Pressure cycle as heat is received at a constant pressure.





Process 1 → 2 : Isentropic Compression

$Q = W + \Delta U \quad , \quad Q = 0$

$W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$

Process 2 → 3 : Reversible constant pressure heating

$Q = W + \Delta U = C_p(T_3 - T_2)$

Process 3 → 4 Isentropic expansion

$Q = W + \Delta U \quad , \quad Q = 0 \Rightarrow W = -\Delta U = -C_v(T_4 - T_3)$

$W = C_v(T_3 - T_4)$

Process 4 → 1 , Reversible Constant Volume heating.

$Q = W + \Delta U \quad , \quad W = 0 \; , \; V = c \Rightarrow Q = \Delta U = C_v(T_1 - T_4)$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{1}{\gamma}\,\frac{T_4 - T_1}{T_3 - T_2}$$

Process $2 \rightarrow 3$ $P = C$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow T_3 = T_2 * \frac{V_3}{V_2} \quad \ldots (1)$$

Process $3 \rightarrow 4$ isentropic

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 * \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\because V_4 = V_1 \Rightarrow T_4 = T_3 * \left(\frac{V_3}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 * \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1} = T_3 * \left(\frac{V_3}{V_2}\right)^{\gamma-1} * \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \ldots (2)$$

Sub. eq(1) in eq (2)

$$T_4 = T_2 * \left(\frac{V_3}{V_2}\right) * \left(\frac{V_3}{V_2}\right)^{\gamma-1} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2 * \left(\frac{V_3}{V_2}\right)^{\gamma} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow T_1 = T_2 * \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{T_2 * \left(\frac{V_3}{V_2}\right)^{\gamma} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} - T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}}{\gamma * \left(T_2\left(\frac{V_3}{V_2}\right) - T_2\right)}$$

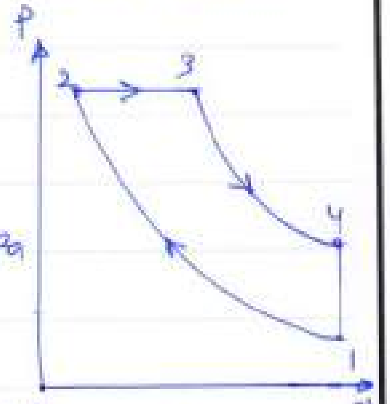
let $r_v = \frac{V_1}{V_2}$ compression ratio

$\beta = \frac{V_3}{V_2}$ cutt off ratio

$$\eta_{th} = 1 - \frac{\cancel{T_2}\left(\beta^{\gamma} \cdot \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}\right)}{\gamma \cancel{T_2}(\beta - 1)} = 1 - \frac{\beta^{\gamma} \cdot \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}}{\gamma(\beta - 1)}$$

$$\therefore \boxed{\eta_{th} = 1 - \frac{(\beta^{\gamma} - 1)}{(\beta - 1)\gamma\, r_v^{\gamma-1}}}$$

Ex.:- An engine uses air as a working substance, at the begining of compression the pressure is 90 kPa and the temp. is 40°C. During the adiabatic compression the volume is reduced to one-sixteenth of it's volume at the begining of compression stroke. Heat is then added at constant pressure until the temp. is 1400°C. The strok is completed by adiabatic expansion until the initial volume is reduced, and the cycle is closed by constant volume process. Find: a- the temp. and at all four corner. b- thermal efficiency.



Sol.:-

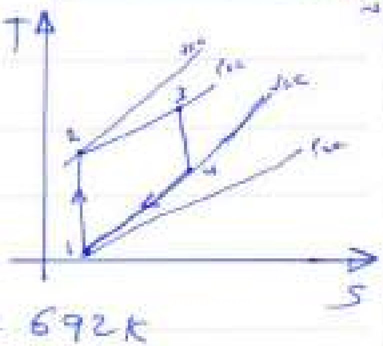
$P_1 = 90$ kPa , $T_1 = 313$ K

$V_2 = \frac{1}{16} V_1 \Rightarrow P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot P_1 = (16)^{1.4} * 90 = 4365$ kPa

$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} * T_1 = (16)^{0.4} * 313 = 949$ K

$P_3 = P_2 = 4365$ kPa ; $T_3 = 1400 + 273 = 1673$ K

$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1}$



$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_3}{T_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 * \left(\frac{T_3}{T_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1} = 1673 * \left(\frac{1673}{949} \cdot \frac{1}{16}\right)^{0.4} = 692\text{K}$$

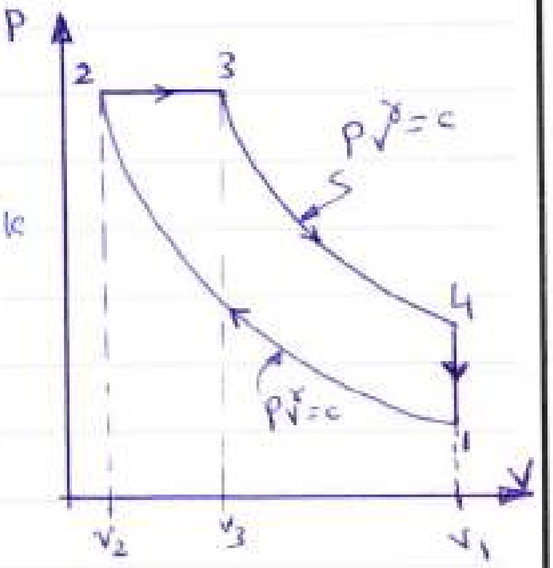
$$P_4 = P_3 \left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 4365 \left(\frac{692}{949}\right)^{\frac{1.4}{0.4}} = 198.6 \text{ kPa}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v (T_4 - T_1)}{C_p (T_3 - T_2)} = 1 - \frac{692 - 313}{1.4(1673 - 949)} = 0.63$$

or 63%

Ex.: A diesel engine has an inlet temperature and pressure of 15°C and 1 bars respectively. The compression ratio is 12:1 and the maximum cycle Temperature is 1100°C. Calculate the air standard thermal efficiency based on the Diesel Cycle.

Sol.:



$$T_1 = 15+273 = 288\ K$$

$$T_3 = 1100+273 = 1373\ K$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r_v)^{\gamma-1},\ \gamma=1.4,\ T_2 = 778\ K$$

Process 2 → 3 $\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$; $P_3 = P_2$

$$\therefore \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1377}{728} = 1.765$$

$$\frac{V_4}{V_3} = \frac{V_4}{V_2}\cdot\frac{V_2}{V_3} = \frac{V_1}{V_2}\cdot\frac{1}{V_3/V_2} = 12 * \frac{1}{1.765} = 6.8$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \Rightarrow \frac{1373}{T_4} = (6.8)^{0.4} \Rightarrow T_4 = 638\ K$$

$$Q_1 = C_p(T_3-T_2) = 1.005(1373-778) = 598\ kJ/kg$$

$$Q_2 = C_v(T_4-T_1) = 0.718(638-288) = 251\ kJ/kg$$

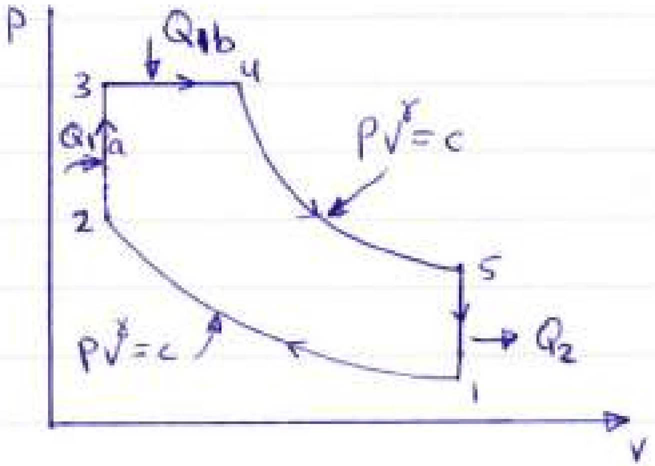
$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{251}{598} = 0.58 = 58\%$$

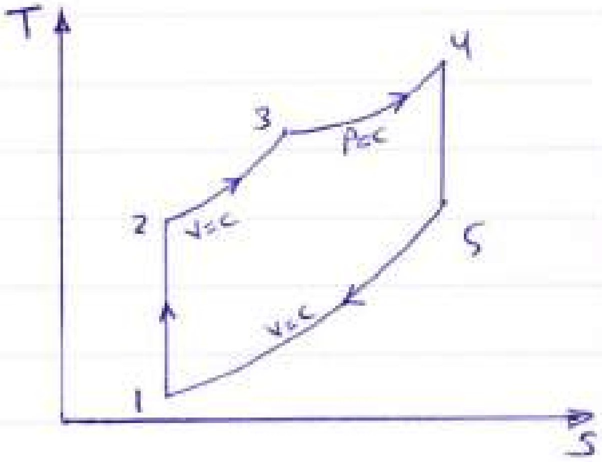
or $\beta = 1.765$; $r_v = 12$ $\Rightarrow \eta_{th} = 1 - \frac{(\beta^{\gamma}-1)}{\gamma*(\beta-1)\,r_v^{\gamma-1}}$

$$\eta_{th} = 1 - \frac{1.765^{1.4}-1}{1.4*(1.765-1)*12^{0.4}} = 0.58 \text{ or } 58\%$$

Dual Combustion Cycle :-

Modern oil engines althoug still Called diesel engines are more closely derived from an engine invented by Ackroy - stuar . The heat is Supplied in two Parts. the first Part at constant Volume and remainder at constant Pressure , hence the name "dual Combustion."





Process: 1 → 2 Adiabatic Compression $PV^{\gamma}=C$

$Q = W + \Delta U$, $Q = 0$, $W = -\Delta U = U_1 - U_2 = C_v(T_2 - T_1)^{+ve}$

Process : 2 → 3 Reversible Constant Volume heating

$V = C$ $W = 0$ $\Rightarrow Q_{1a} = C_v(T_3 - T_2)$

Process : 3 → 4 Reversible Constant Pressure heating (Q_{1b})

$P = C$; $Q_{1b} = W + \Delta U = h_4 - h_3 = Q_{1b} = C_p(T_4 - T_3)$

Process : 4 → 5 Adiabatic expansion $PV^{\gamma}=C$

$Q = W + \Delta U$, $Q = 0 \Rightarrow W = -\Delta U = -C_v(T_5 - T_4)$

$= C_v(T_4 - T_5)$

Process 5 → 1 Reversible Constant Volume Cooling (Q_2)

$V = C \Rightarrow W = 0 \Rightarrow Q_2 = \Delta U = C_v(T_5 - T_1)$.

$r_v = \frac{V_1}{V_2}$ compression ratio

$\alpha = \frac{P_3}{P_2}$ = ratio of pressure during heat adding at constant volume.

$\beta = \frac{V_4}{V_3}$ = ratio of volume during heat adding at constant pressure.

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2}{Q_{1a}+Q_{1b}}$$

$$\eta_{th} = 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$\eta_{th} = 1 - \frac{(\alpha\beta^{\gamma} - 1)}{[(\alpha - 1) + \gamma \cdot \alpha(\beta - 1)]\, r_v^{\gamma-1}}$$

if $\alpha = 1$ (heat added only at $P = c$)

$$\eta_{th} = 1 - \frac{\beta^{\gamma} - 1}{r^{\gamma-1} \cdot \gamma(\beta - 1)}$$

if $\beta = 1$ (heat added only at $V = c$)

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}$$

Ex.:- An oil engine takes at 1.01 bar, 20 °C and the max. pressure is 69 bar. The compression ratio is 18. Calculate the air standard thermal efficiency based on the dual combustion cycle. Assuming that the heat added at constant volume is equal to the heat added at constant pressure.

Sol.

$$\eta_{th} = 1 - \frac{Q_2}{Q_{1a} + Q_{1b}}$$

$$= 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma (T_4 - T_3)}$$

$P_1 = 1.01$ bar, $T_1 = 298$ k

$$T_2 = T_1 * \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 298 * (18)^{0.4} = 931 \text{ k}$$

$$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} * P_1 = 57.8 \text{ bar}$$

$P_3 = 69$ bar $\quad \frac{P_2 \cancel{V_2}}{T_2} = \frac{P_3 \cancel{V_3}}{T_3} \Rightarrow T_3 = \frac{P_3}{P_2} \cdot T_2$

$$T_3 = \frac{69}{57.8} * 931 = 1112 \text{ k}$$

$P_4 = P_3 = 69$ bar, $Q_{a} = Q_{1b}$

$c_v(T_3 - T_2) = c_p(T_4 - T_3)$; $T_4 = \frac{1}{\gamma}(T_3 - T_2) + T_3 = 1241$ k

$(T_5 / T_4) = (V_4 / V_5)^{\gamma - 1} = (V_4 / V_1)^{\gamma - 1}$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_3} \cdot \frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{T_4}{T_3} \cdot 1 \cdot \frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{1241}{1112} \cdot 1 \cdot \frac{1}{18}\right)^{0.4} = 408 \text{ k}$$

$$\therefore \eta_{th} = \frac{408 - 293}{(1112 - 931) + 1.4(1241 - 1112)} = 68.1\%$$